Lecture Notes

Introduction to Strongly Correlated Electron Systems

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Introduction to strongly correlated electron systems

I. Introduction

Brief summary of electrons in solids, origin of strong electron correlations

II. Classes of strongly correlated electron systems

(a) Transition metal compounds: 3d-electrons

- Hubbard model, Mott insulator, metal-insulator transition
- Spin, charge, and orbital degrees of freedom and ordering phenomena, selected materials
- Pressure effect on the ground state properties of transition metal compounds

(b) Heavy fermion systems: 4f (5f) – electrons

- Landau Fermi-liquid model, Kondo effect, heavy fermion systems, non-Fermi liquid, quantum phase transitions, selected materials
- Pressure effect on the ground state properties of heavy fermion compounds

(c) Nanoscale structures:

- Quantum confinement, unusual properties for potential applications

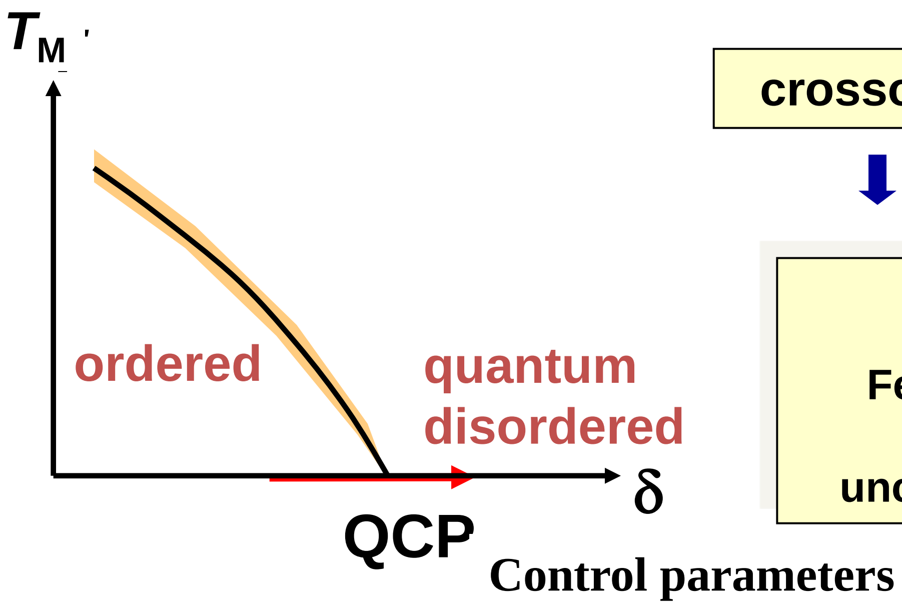
III. Summary and open discussion

Heavy fermion metallic systems

Intermetallic Ce (4f), Yb (4f) and U(5f) - compounds

local moments → **itinerant moments**

increasing hybridization between localized states and conduction electrons



crossover

⬇

<u>unusual ground states</u>:

Fermi-liquid, Non-Fermi-liquid, heavy fermions, new magnetic and unconventional superconducting states

Physical picture: crossover magnetic ↔ nonmagnetic

Interaction between the Spins of conduction electrons with impurity spins

⇒ Spin correlations

Strong resonace scattering of conduction electrons by the local moments

⬇

Formation of an (Abrikosov-Suhl) resonance at E_F of width $k_B T_K$

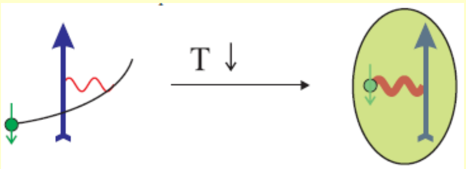
⬇

Logarithmic increase of ρ below T_K

➡

***$T \ll T_K$*:**

a) impuity **magnetic Moment is screend by the** Spins of conduction electrons**. This leads** to formation of a **local Singlet state**



b) Energy lowering due to formation of a Kondo-state:

$$k_B T_K = D e^{-\frac{1}{|J| N(E_F)}}$$

⬇

Crossover:

magnetic ↔ nonmagnetic

weak ↔ strong coupling

Kondo effect in concentrated alloys

Kondo-lattice systems (heavy fermions)

Properties of Kondo-Lattice systems (Heavy Fermions)

Electrical resistivity:
deviation from single ion behavior

Periodicity of the lattice

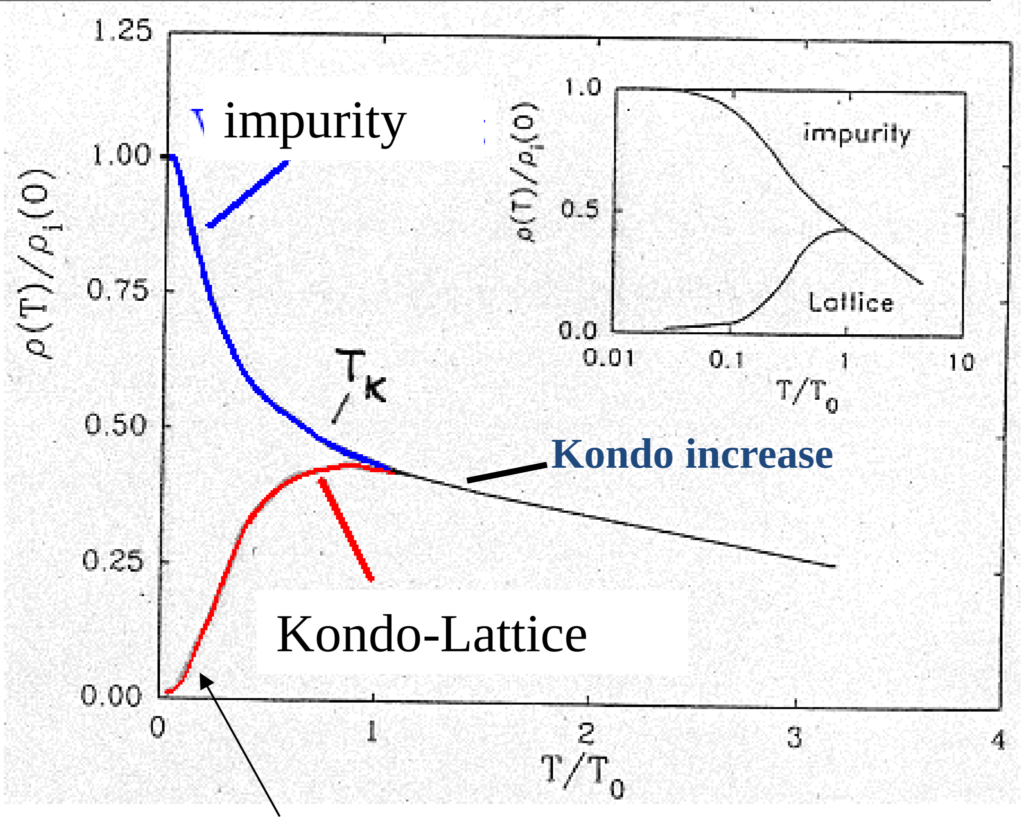
⬇

Coherent scattering of conduction electrons on magnetic impurities

⬇

resonace type increase of the density of state at Fermi level.

Formation of an Abrikosov-Suhl resonance at E_F



T→ 0:
$\rho(T) = \rho_0 + AT^2$
(Fermi-liquid state)

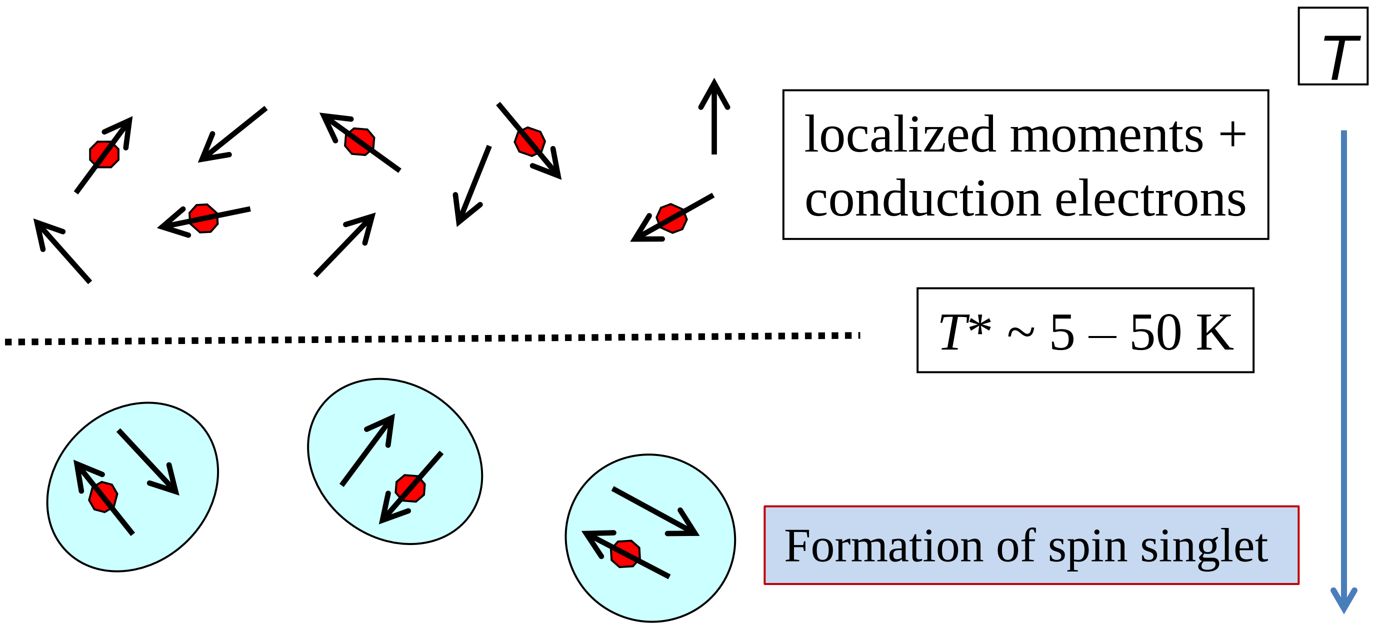
$A \propto [D(E_F)^2]$
(electron-electron interaction)

Kondo-lattice systems (heavy fermions)

Lattice of certain *f*-electrons (most Ce, Yb or U) in metallic environment Ce^{3+}: $4f^1$ (J = 5/2), Yb^{3+}: $4f^{13}$ (J = 7/2)

partially filled inner 4f/5f shells → localized magnetic moment

CEF splitting → effective S=1/2



Kondo-lattice systems (heavy fermions)

characeristic temperature *T**

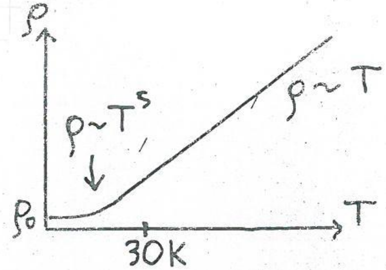
***T* >>*T**:** local moment behavior

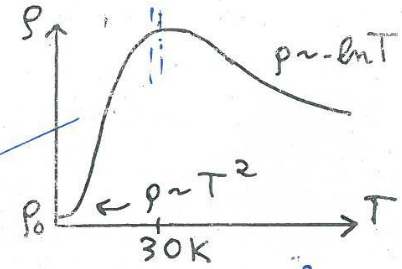
***T* << *T**:** nonmagnetic heavy fermion liquid (Fermi liquid grond state)

See board!

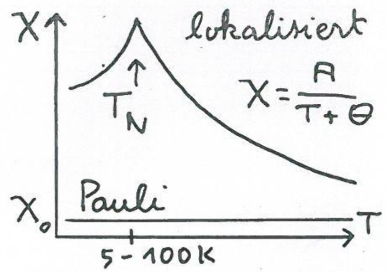
Properties of heavy fermion systems

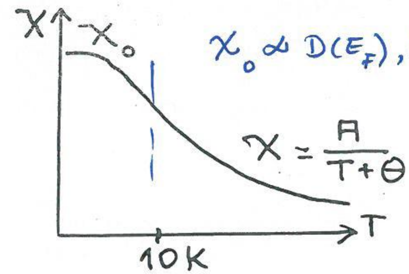
normal metal | heavy fermion system



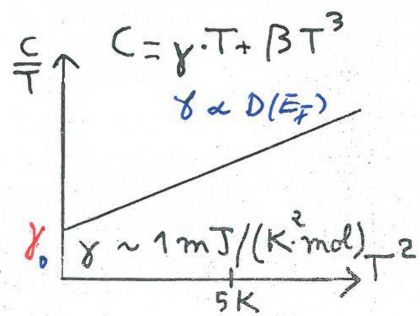


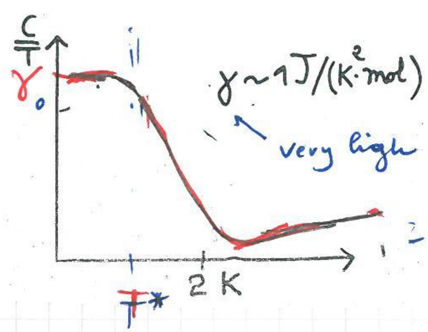
$\rho = \rho_0 + AT^2$, A very high
$A \sim \gamma^2$





$\chi_0 \propto D(E_F)$, high!





Characteristic Temp. T^*
(analog to T_K)

$T \gg T^*$: local moment behavior

$\chi(T) \to$ Curie-Weiss
$\rho(T) \to -\ln T$ (Kondo!)

$T \ll T^*$: nonmagnetic heavy fermion liquid
$\Rightarrow$ FL ground state with large $D^*(E_F)$

$\chi(T) \propto m^* \propto \frac{1}{T^*}$

$\gamma(T)$, $\gamma_0 \propto m^* \propto \frac{1}{T^*}$

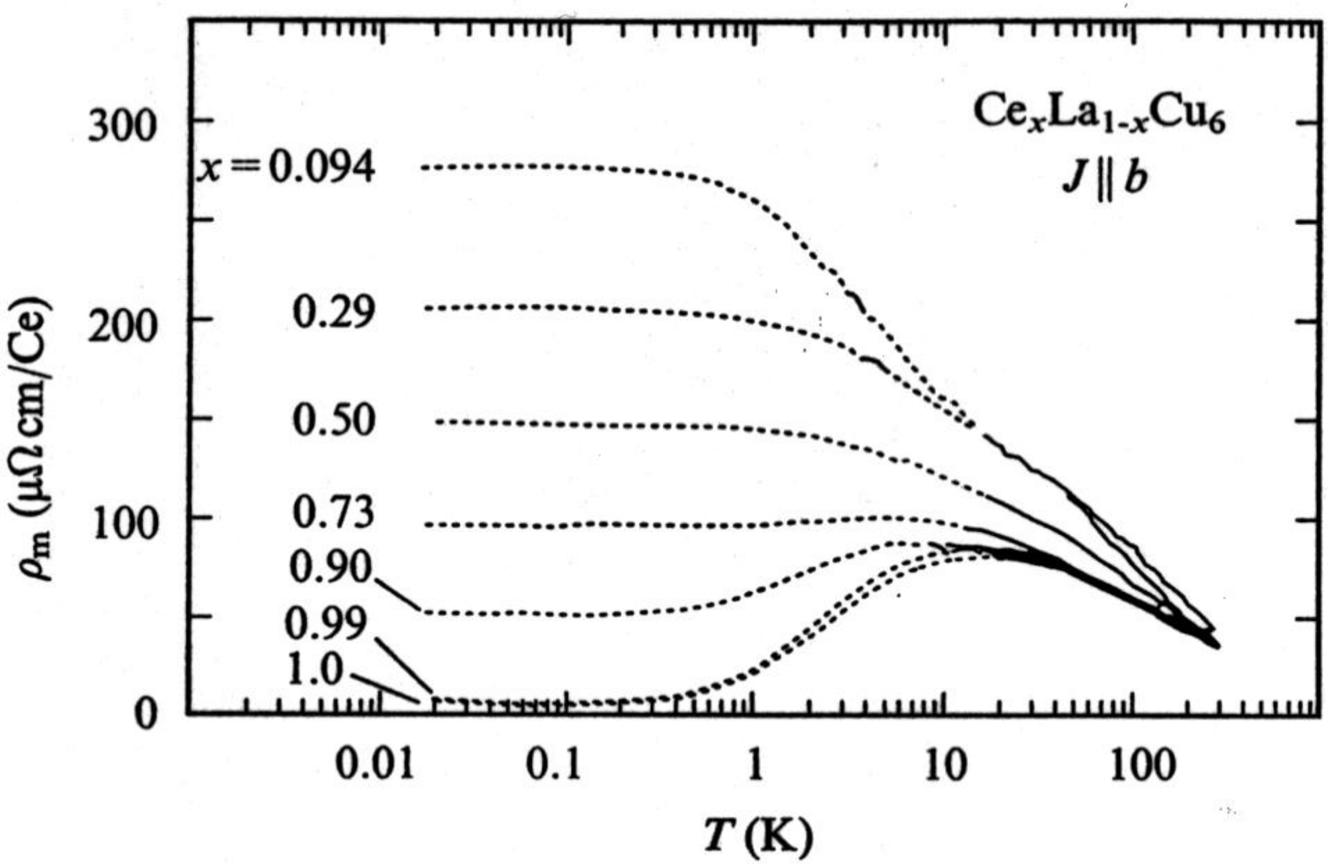
~~$\rho(T) \propto \rho_{e-e}(T)$~~ ~

$\rho(T) \propto AT^2$, $A \sim \gamma_0^2$

Wilson's ratio $R \simeq 1$

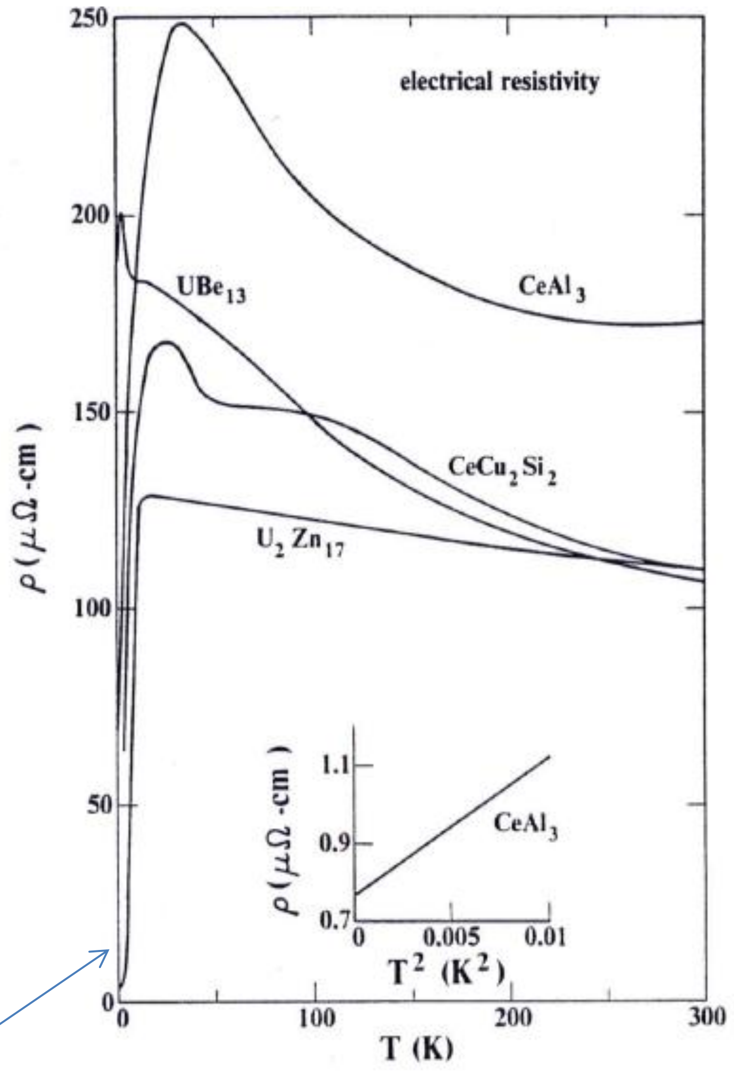
→ show some examples.

Development of coherent scattering in Ce-based alloy



Onuki and Komatsubara (1987)

Kondo-Lattice, heavy fermion systems

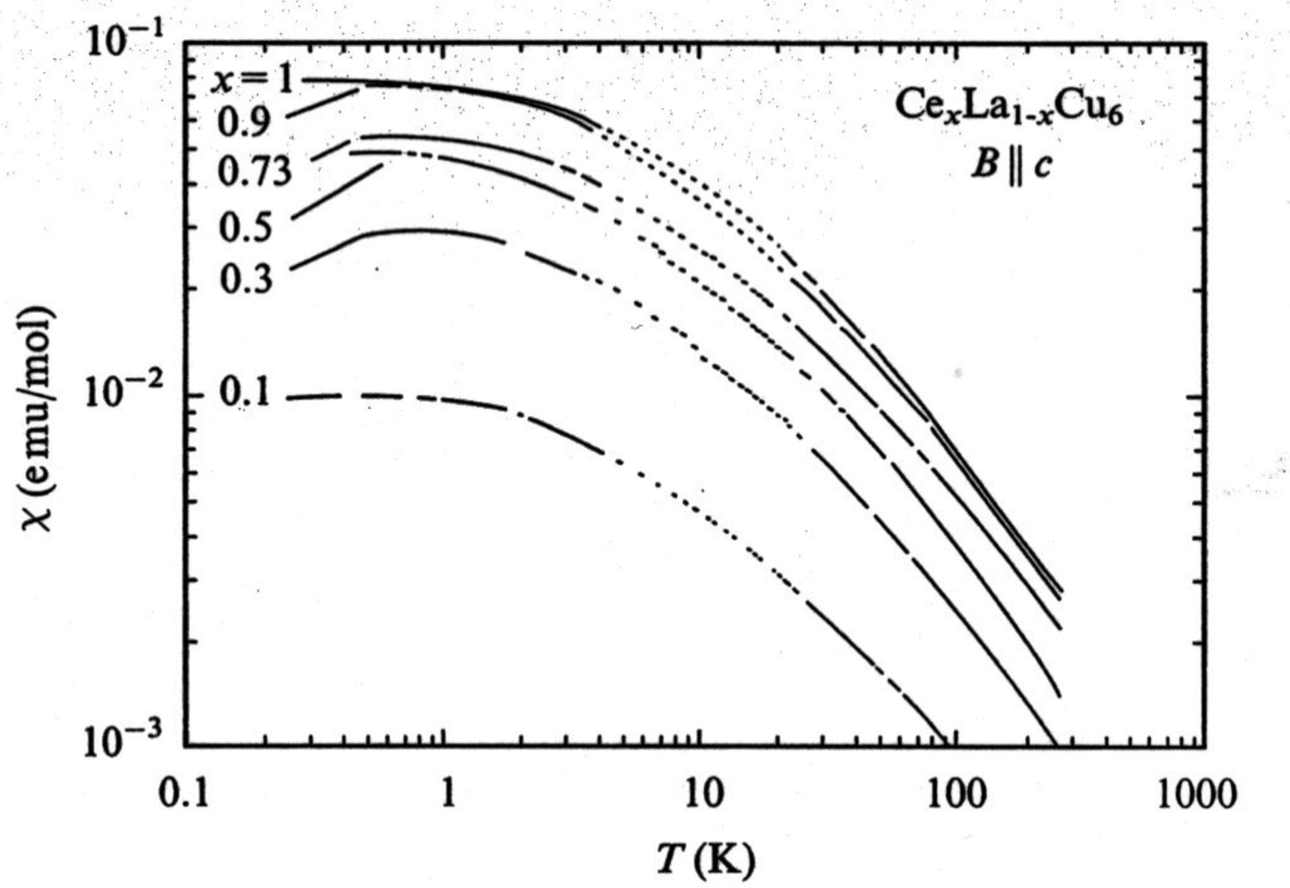


Electrical resistivity

After Fisk, Ott, Rice & Smith 86

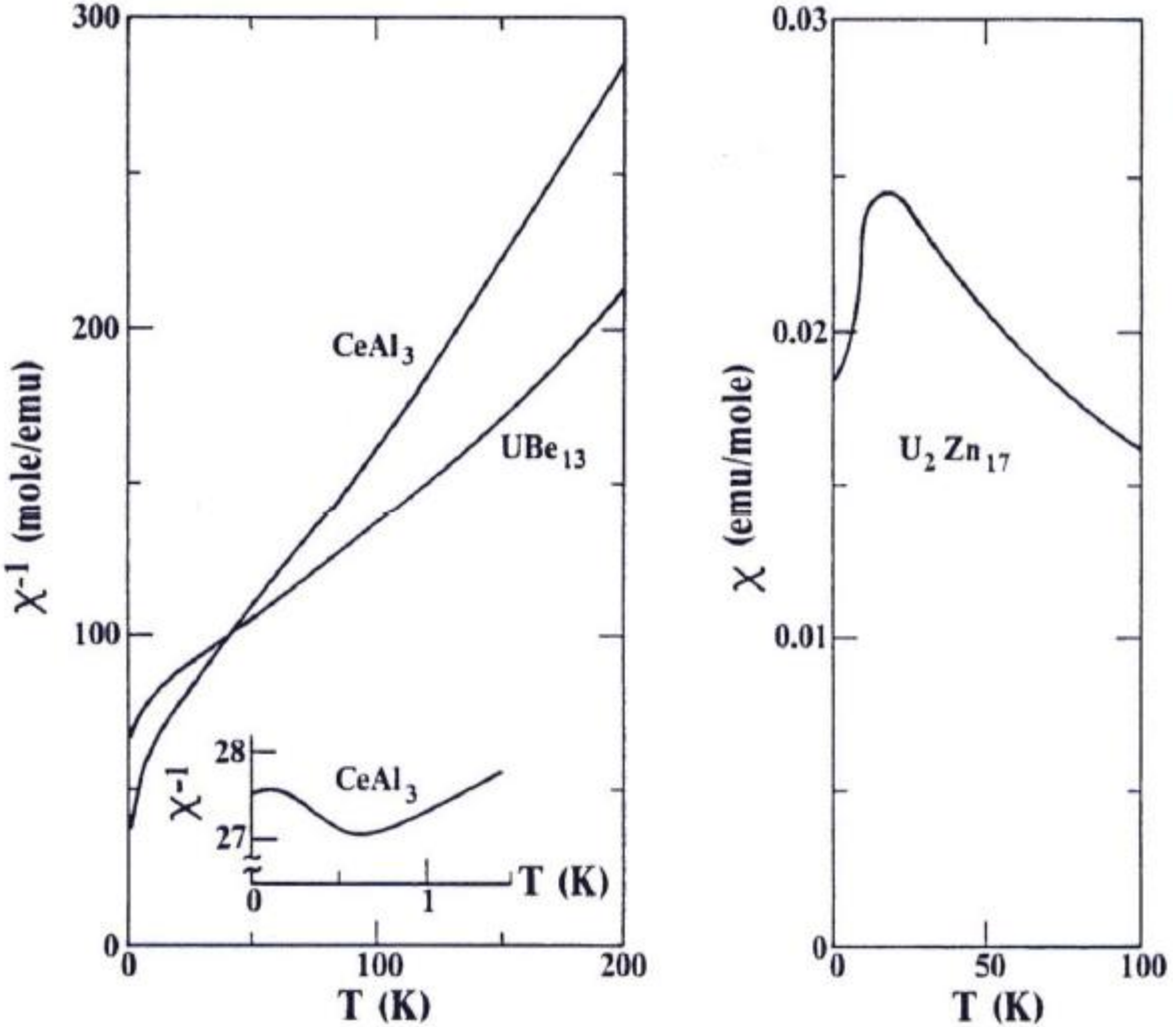
Coherent heavy fermions

Disappearence of the local moments at low temeratures



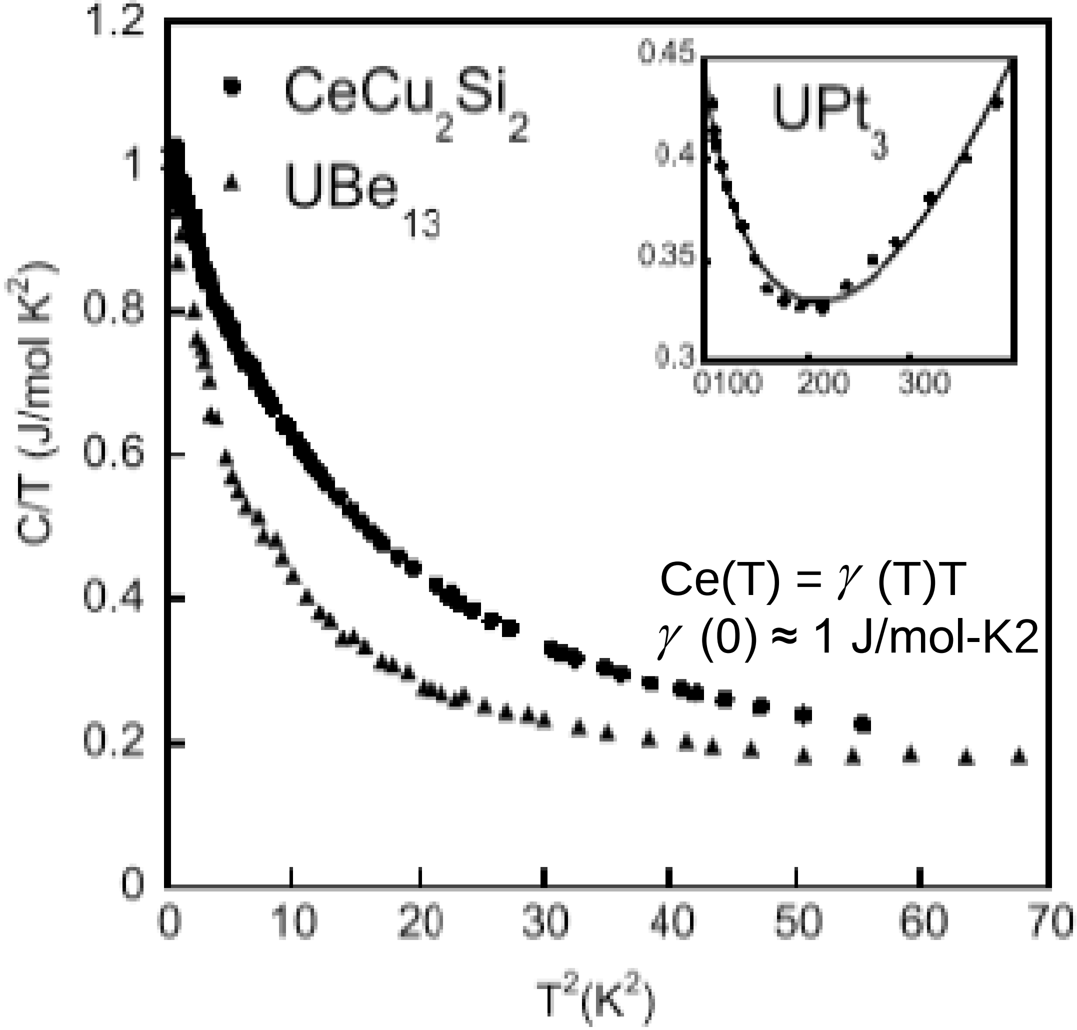
Onuki and Komatsubara (1987)

magnetic susecptibility



After Fisk, Ott, Rice & Smith 86

C/T = γ vs T2 for CeCu2Si2, UBe13, and UPt3 ⟹
very high γ (effective mass!)

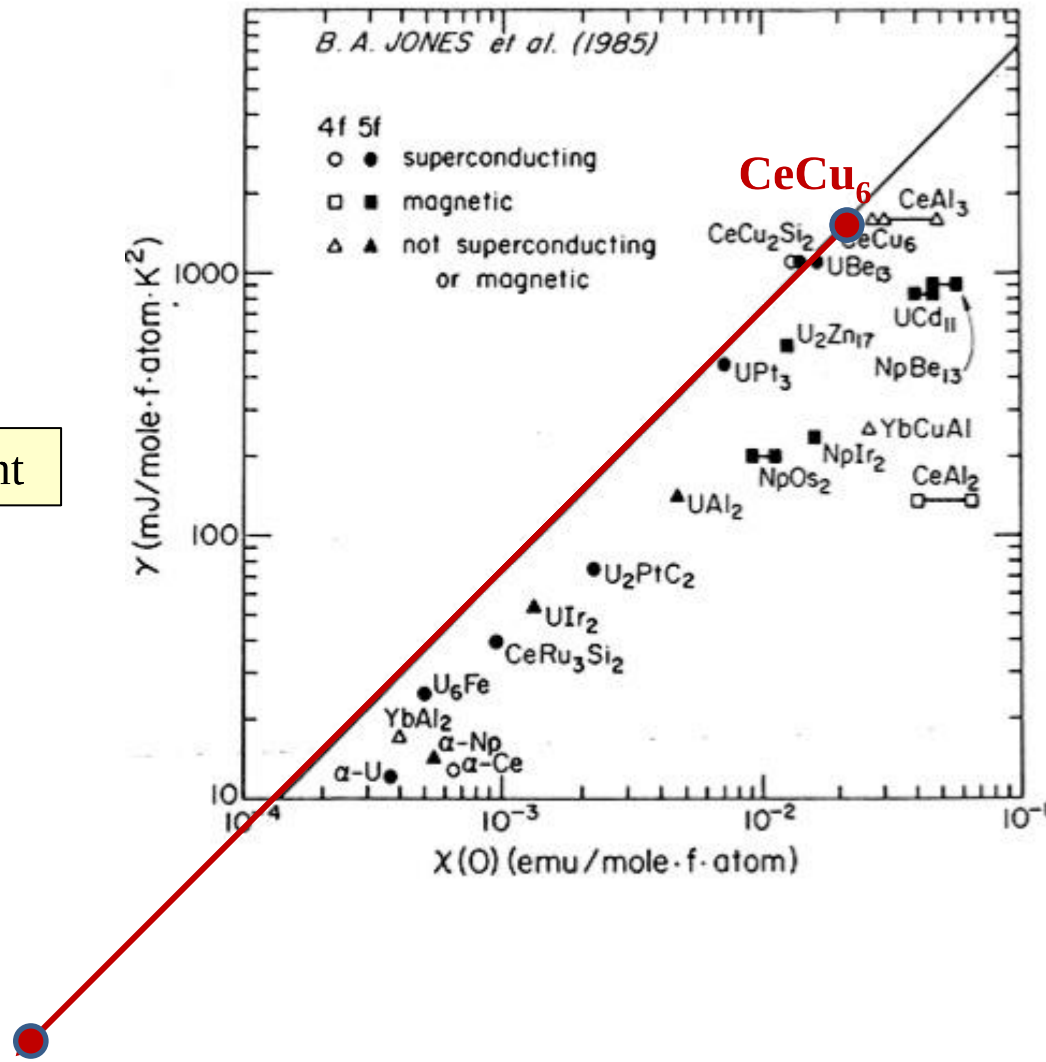


Ce(T) = γ (T)T
γ (0) ≈ 1 J/mol-K2

Wilson s Ratio: $\gamma / T \approx$ constant

γ $CeCu_6$ ~ 1000 mJ/mol/K^2

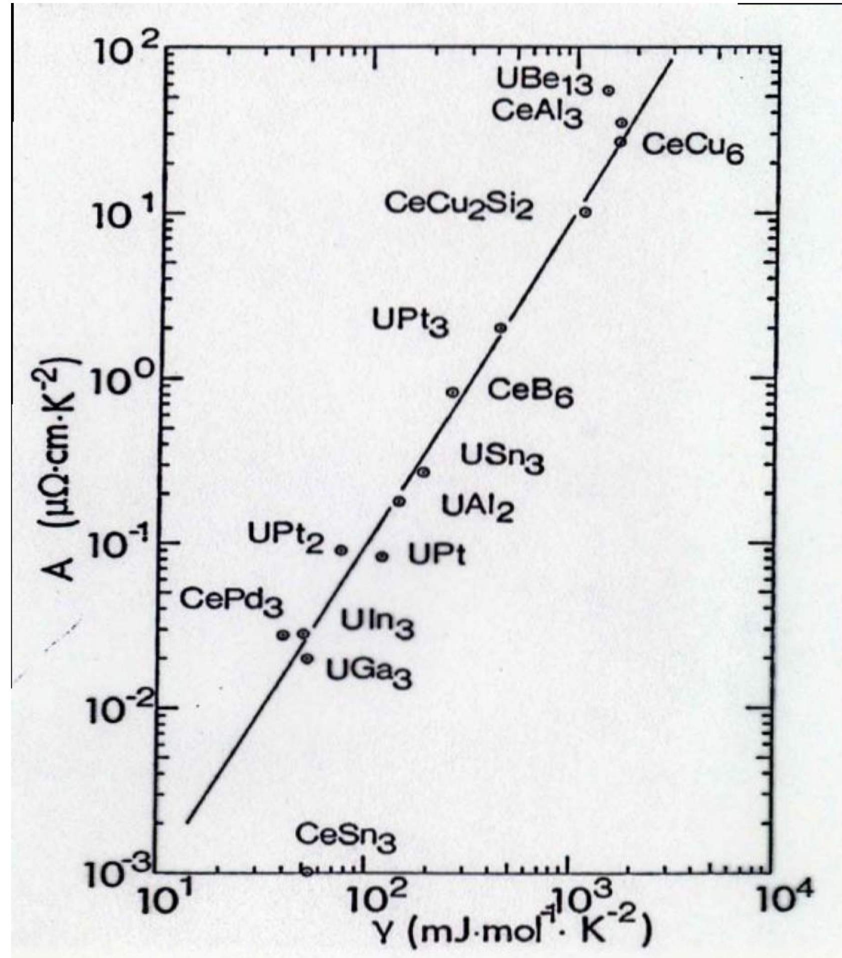
γ Cu ~ 1 mJ/mol/K^2



electron-electron scattering

Kadowaki-Woods plot (1986)

$$\frac{A}{\gamma^2}$$ is constant and material-independent



Observed for a large number of heavy fermion systems

Theoretical description of heavy fermions
Kondo-lattice model

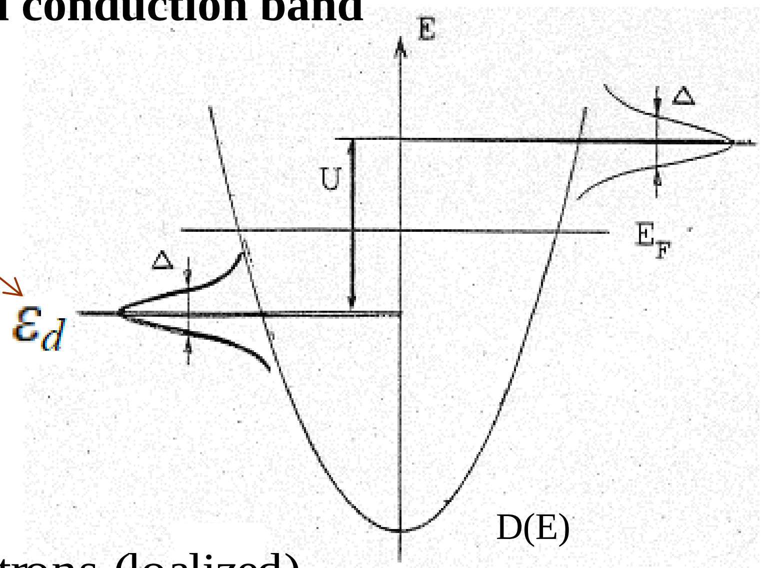
generalizing the single impurity Anderson model to a lattice of localized f orbitals hybridizing with conduction band ⟹ Anderson lattice model

The Anderson model (961)

hybridization between mpurity level and conduction band

⇒ magnetic impurity d (or f) level exhibits finite life time ⇒ finite width Δ

$\Rightarrow \Delta = \pi V_k^2 D(E_F)$



conduction s electrons d electrons (loalized)

$$H_A = \sum_{k\sigma} \varepsilon(k) c^\dagger_{k\sigma} c_{k\sigma} + \sum_{\sigma} \varepsilon_d c^\dagger_{d\sigma} c_{d\sigma} + U n_{d\uparrow} n_{d\downarrow} +$$

$$\sum_{k\sigma} V_k c^\dagger_{k\sigma} c_{d\sigma} + V_k^* c^\dagger_{d\sigma} c_{k\sigma}$$

Coulomb repulsion between d electros

hybridization between local d electrons

and conduction s electrons (Δ)

Theoretical description of ~~km~~ heavy fermions

Kondo-lattice model: generalizing the single impurity Anderson model

(periodic Anderson model) to a lattice of localized f orbitals f_i

$$\Rightarrow \quad \mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_k c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \epsilon_f \sum_{i\sigma} f^{+}_{i\sigma} f_{i\sigma} + U \sum_i n^f_{i\uparrow} n^f_{i\downarrow}$$

dispersive band, conduction elect. — flat f-level — Coulomb repulsion between f electrons

$$+ V \sum_{i\sigma} \left(f^{+}_{i\sigma} c_{i\sigma} + h.c. \right)$$

hybridization between f and conduction band.

local f moments if: U is large and ϵ_f is negative

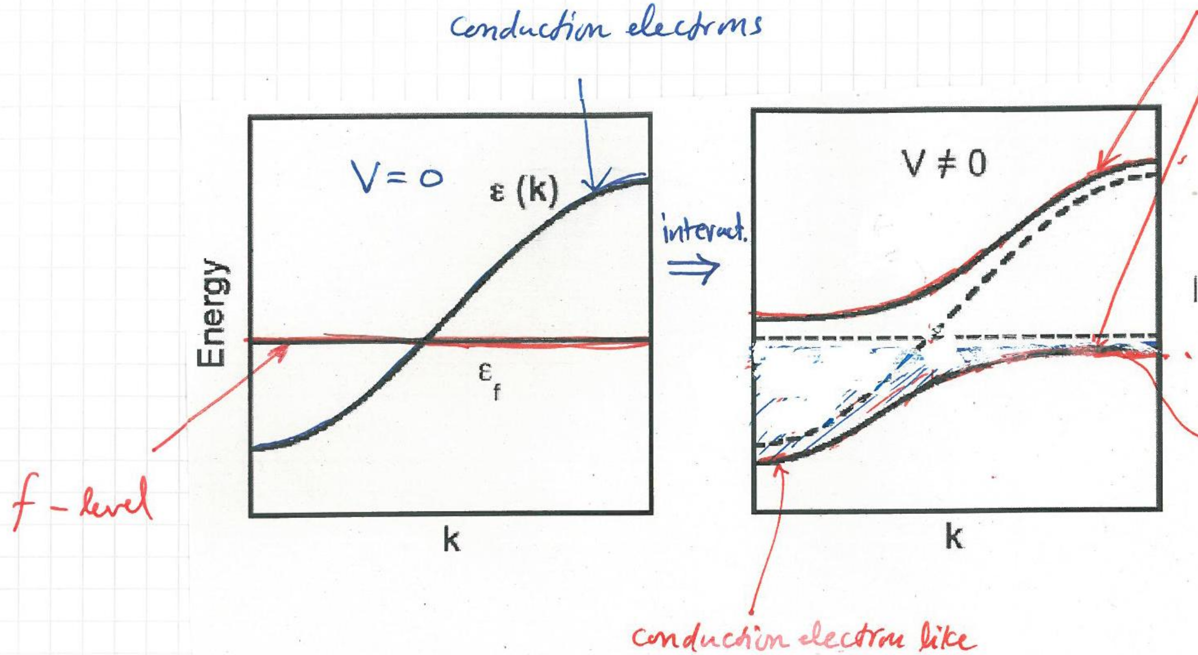
using Schrieffer-Wolff transformation $\Rightarrow$ Kondo lattice model:

$$\Rightarrow \quad \mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + J \sum_i \vec{s}_i \cdot \vec{S}_i$$

$$J = 2V^2 \left(\frac{1}{|\epsilon_f|} + \frac{1}{\epsilon_f + U} \right) > 0$$

Kondo coupling.

Periodic Anderson model (Kondo lattice model)



Effect of hybridization:

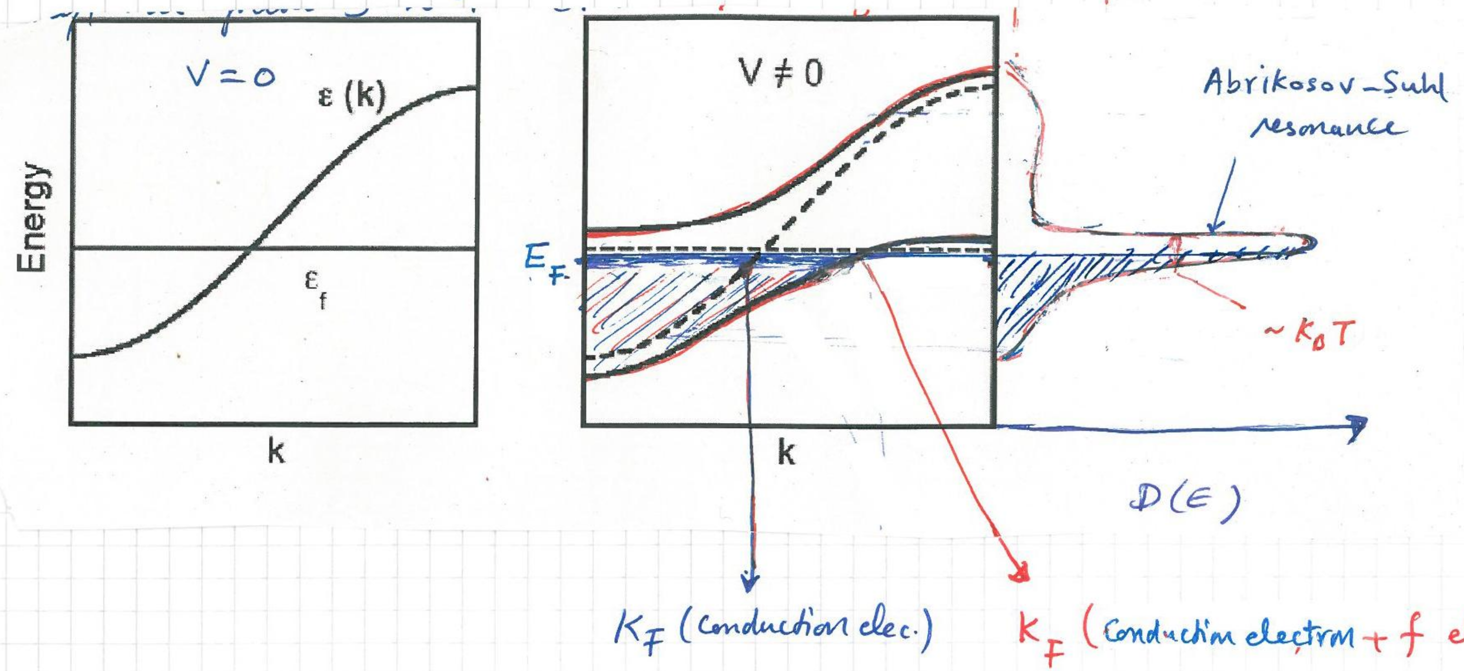
f - level splits due to hybridization with conduction band

⇓

formation of two quasiparticle bands

f-electron like near E_F

Consequences of hybridization between f and conduction band



* a FL ground state can be expected if the local moments are screened by a lattice generalization of the Kondo effect. The onset of Kondo screening at $T \approx T_K$

* the resulting FL, formed below a coherence temperature T_{coh}, $T_{coh} < T_K$

⇒ coherent scattering of conduction electron by the local f moments

⇒ resonance type increase of the density of states at E_F

⇒ formation of an Abrikosov–Suhl resonance at E_F with width $\sim k_B T_K$

⇒ high density of states ⇒ high effective mass m^* !

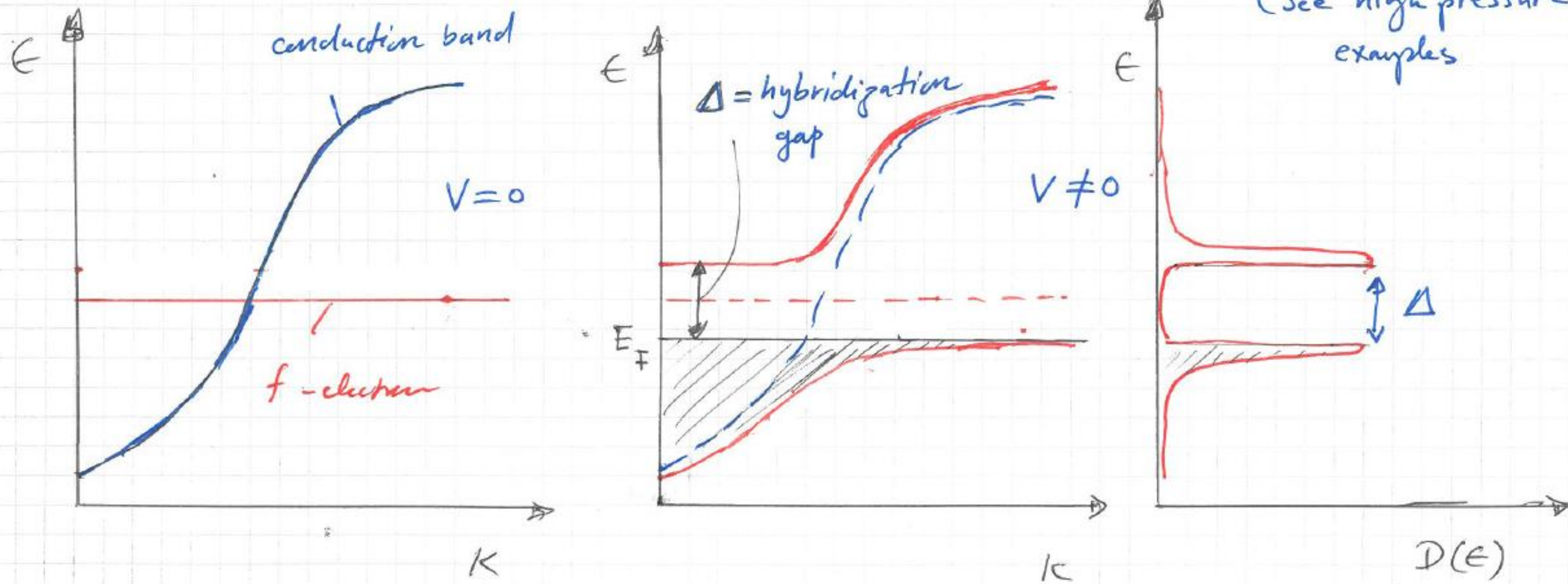
Kondo insulators

Simple picture:

e.g. SmB_6

UNiSn

(see high pressure examples)



Consequences of hybridization:

⇒ f-electrons participate to the Fermi surface (FS)

⇒ large volume of the Fermi surface

$$\boxed{V_{FS} = 4\pi^3 (n_c + n_F)}$$, n_c = nº of conduction ele. ; n_f = " f-electrons

further comments:

* $T > T_{coh}$ ⇒ local moments exsist ⇒ system has a Fermi volume described by cond. electrons ⇒ $V_{FS} = \frac{4\pi}{3} n_c$ (usually small)

* $T \sim T_{coh}$: The FS fluctuates strongly ⇒ resistivity is enhanced

T_{coh} ⟺ resistivity maximum (experimental point of view)

* $T < T_{coh}$: ⇒ screening of local moments:

- FL behavior in the Kondo lattice competes with interaction between the local moments (indirect RKKY interaction) ⇒ long range magnetic order
- other competitors of the FL may be magnetic frustration, spin glass and spin liquid

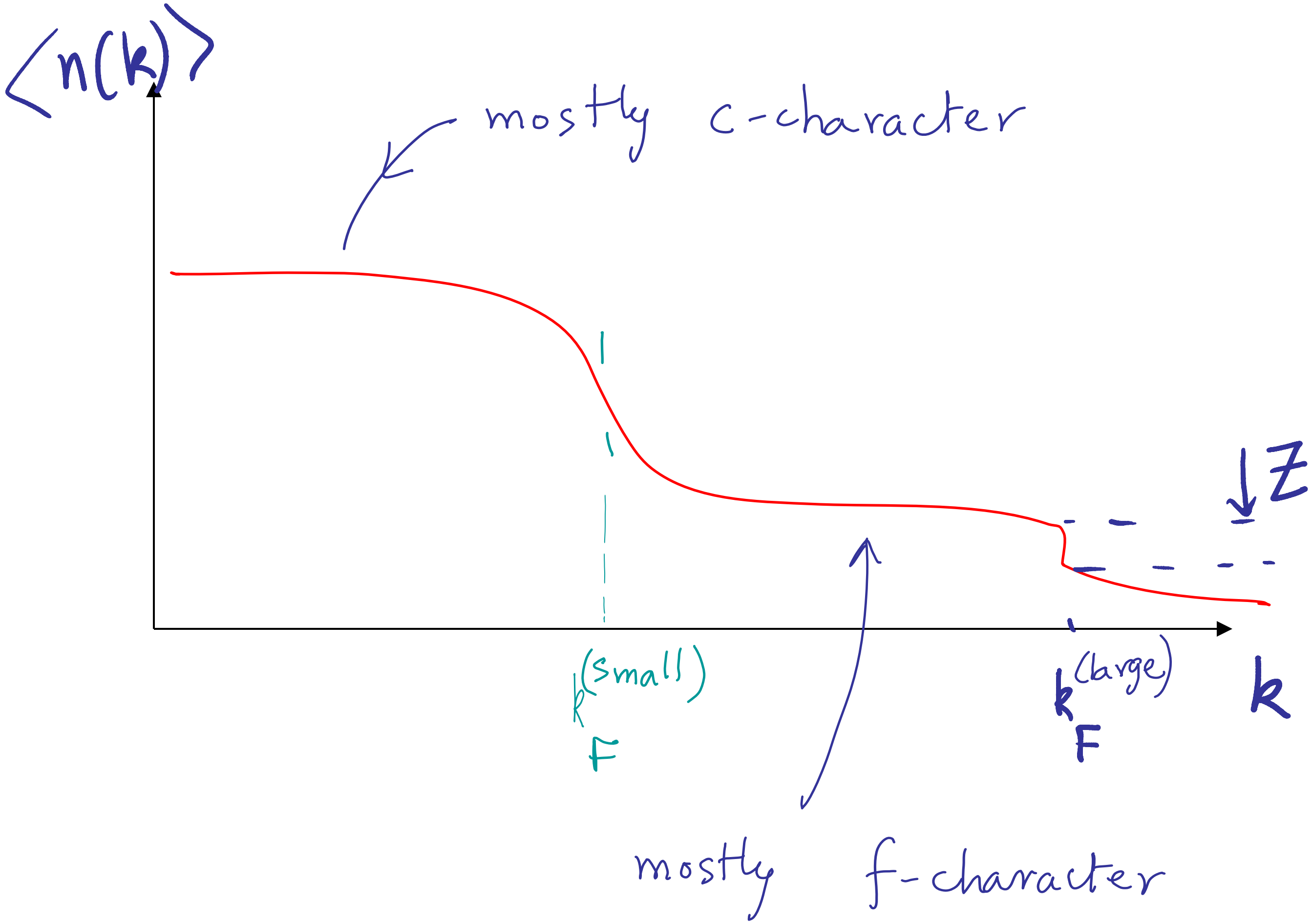
Experimental support for description of heavy fermion metals by Landau Fermi Liquid Theory

-The volume of the Fermi surface includes the *f* electrons.

-The measured quasiparticle mass accounts for the enhanced specific heat.

Both these observations confirm the success of Fermi liquid theory.

Momentum distribution



The *f* electrons participate to the Fermi surface

Example: UPt3

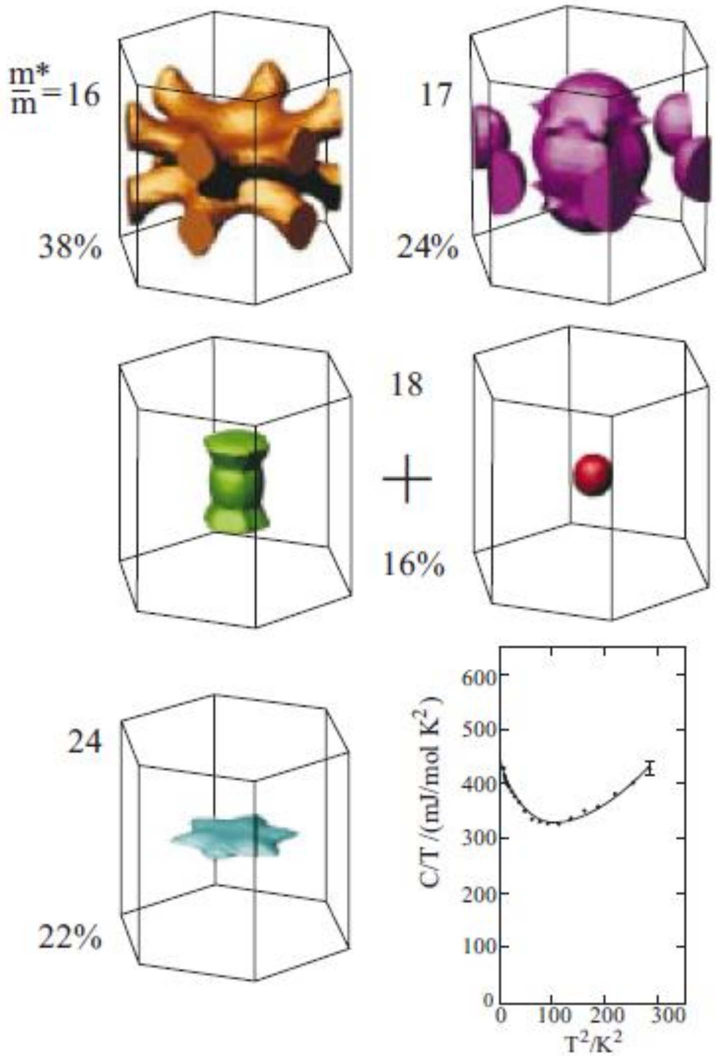
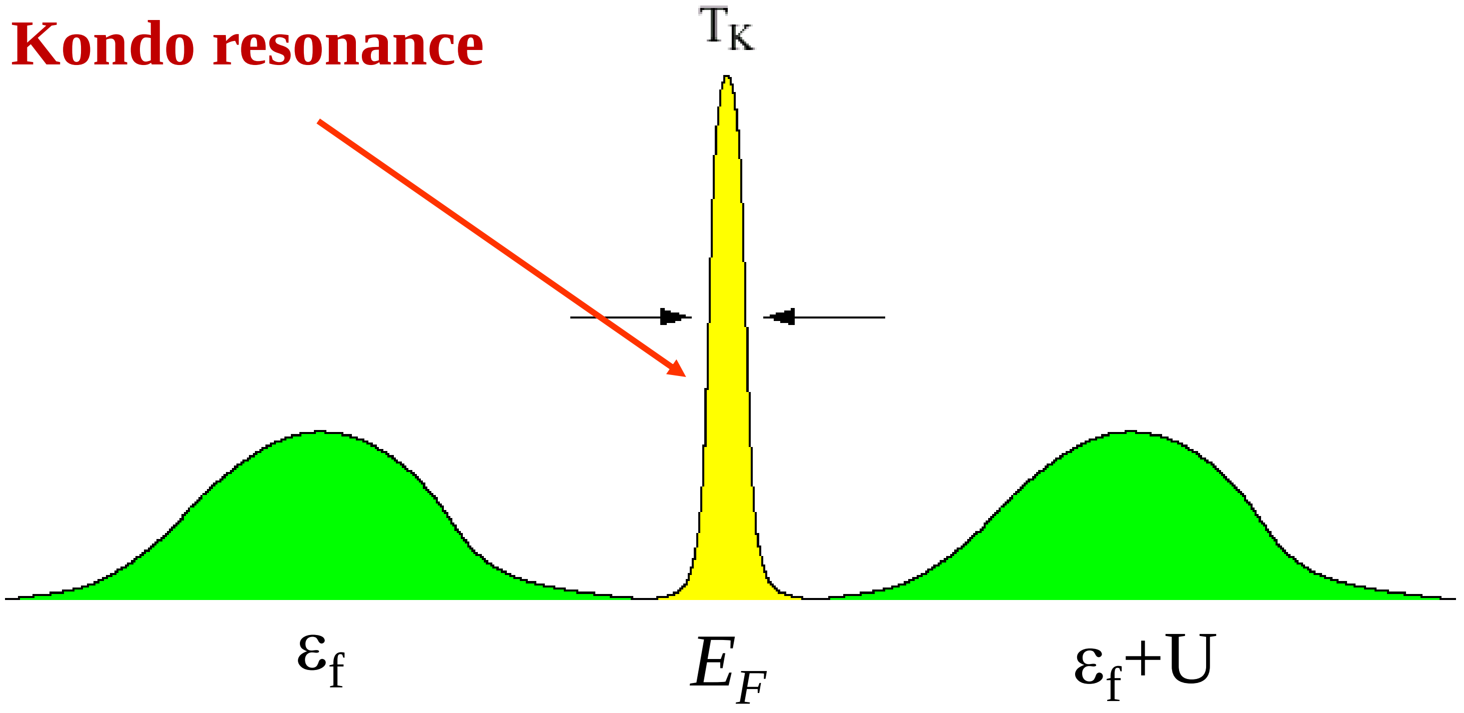


Figure 7: The consistency of the Fermi liquid description has been demonstrated in UPt3. The Fermi surface sheets (from Julian and McMullan 1998) and Effective mass of the quasiparticles have been mapped out by de Haas van Alphen measurements (Taillefer and Lonzarich 1988). They confirm that the 5*f3 electrons* are absorbed into the Fermi liquid and that the quasiparticle masses are consistent with the mass enhancements measured in specific heat (after Stewart *et al.* 1984). The percentages reflect the contribution from quasiparticles on each sheet to the total specific heat.

can we observe the Kondo resonance experimentally ?

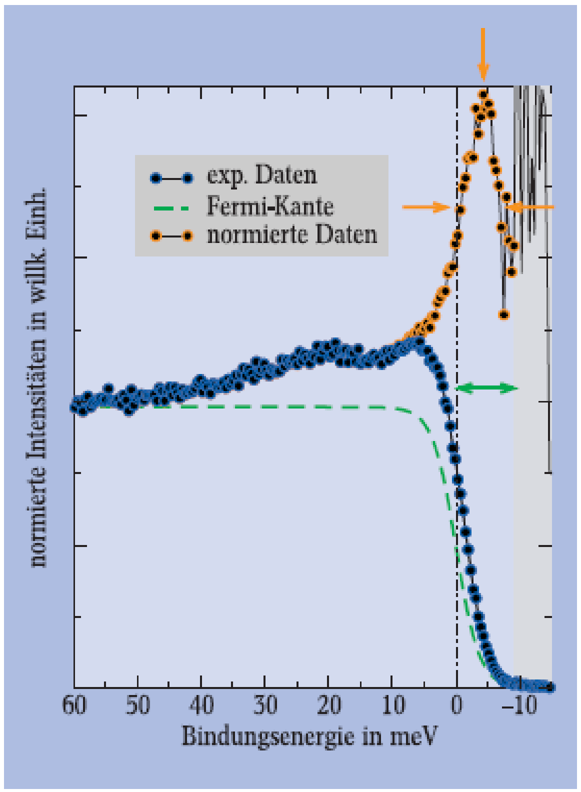


Formation of an (Abrikosov-Suhl) resonance at E_F of width $k_B T_K$

High-resolution photoemission spectroscopy of CeCu2Si2

Reiner et al PRL (2001)

Kondo resonance peak at E_F
Theoretical prediction



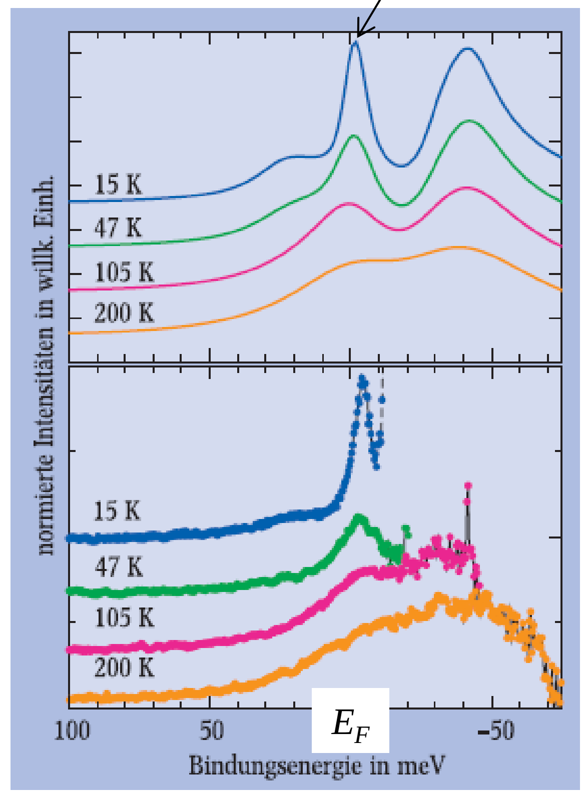


Abb. 8:
Hochaufgelöstes Photoemissionsspektrum an $CeCu_2Si_2$ bei $T = 11$ K in der Nähe der Fermi-Energie, vor und nach einer Normierung auf die Fermi-Verteilung. Nach der Normierung wird die Existenz der scharfen Kondo-Resonanz deutlich, deren Breite und Energie nur wenige Millielektronenvolt beträgt.

Evolution of the Kondo resonance peak at E_F at low temperatures

Kondo Effect in Mesoscopic Systems

Nanotechnology has rekindled interest in the Kondo effect, one of the most widely studied phenomena in condensed-matter physics

Revival of the Kondo effect

Leo Kouwenhoven and Leonid Glazman,
Physics World (January 2001) 33-38.

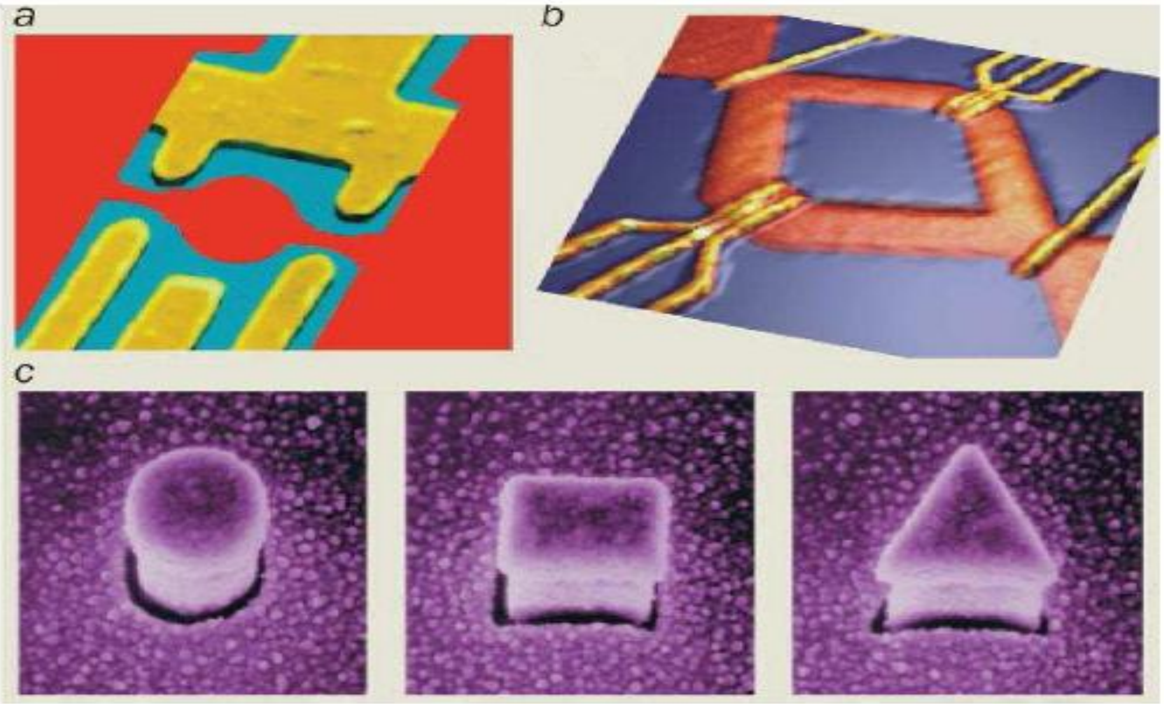

ASAHI SHIMBUN

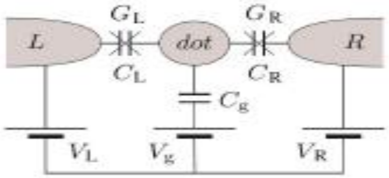
WHY would anyone still want to st
physical phenomenon that was di
ered in the 1930s, explained in the 1
and has been the subject of num
reviews since the 1970s? Althoug
Kondo effect is a well known and w
studied phenomenon in conde
matter physics, it continues to ca
the imagination of experimentalist
theorists alike.

The effect arises from the interac
between a single magnetic atom,
as cobalt, and the many electrons
otherwise non-magnetic metal. Su
impurity typically has an intrinsic a
lar momentum or "spin" that inte
with the electrons. As a result, the m
matical description of the system is

ve as a single entity. Indeed, super-
uctivity is a prime example of a
-electron phenomenon.
her metals, like copper and gold,
in conducting and have a constant
resistance, even at the lowest ac-
le temperatures. The value of the
emperature resistance depends on
umber of defects in the material.
ng defects increases the value of
saturation resistance" but the char-
of the temperature dependence
ins the same.
wever, this behaviour changes dra-
cally when magnetic atoms, such as
lt, are added. Rather than satur-
, the electrical resistance increases
e temperature is lowered further

Quantum dots – mesoscopically fabricated, tunneling of single electrons from contact reservoir controlled by gate voltage





Regions that can hold a few hundred electrons!

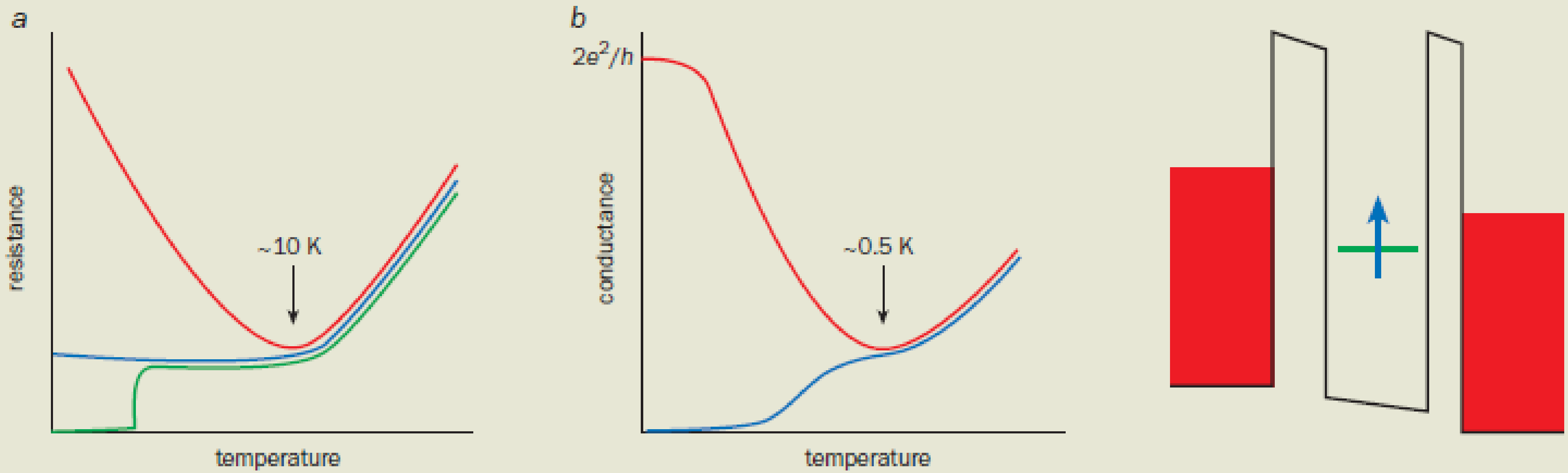
Can drive a current through these!

This *is* Nano!

How is conduction related to the Kondo effect?

Physics World (January 2001) 33-38

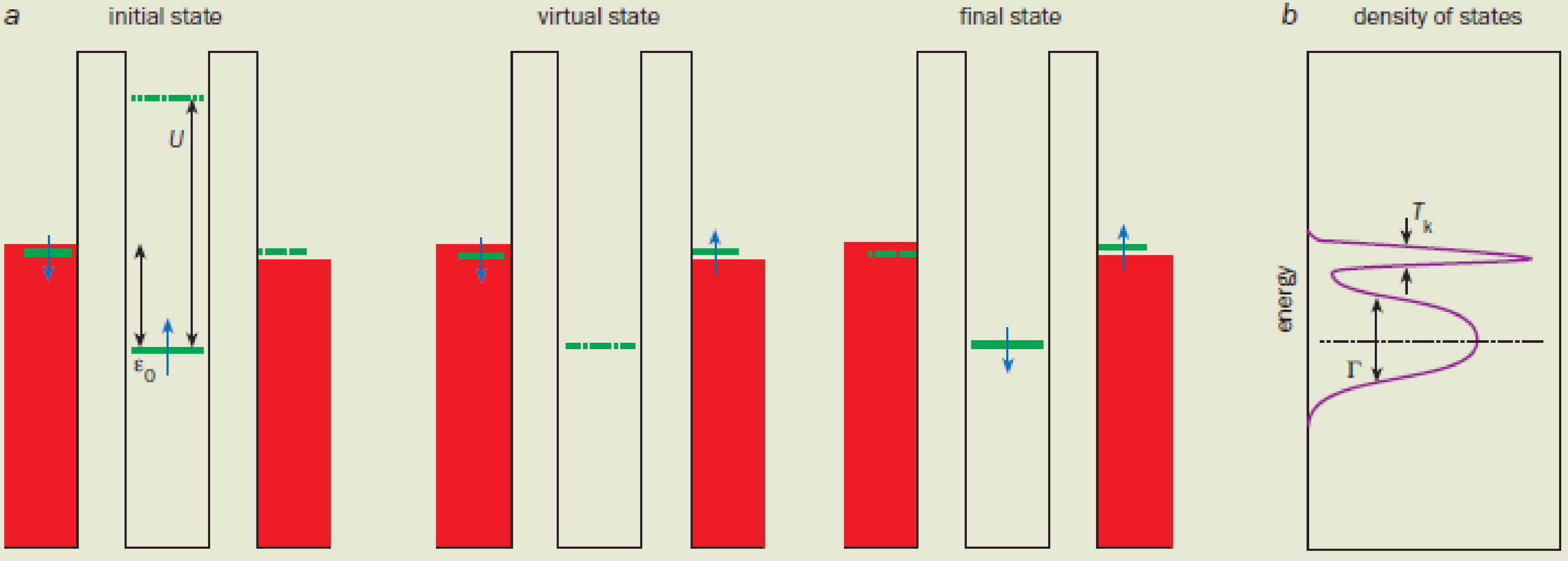
1 The Kondo effect in metals and in quantum dots



(*a*) As the temperature of a metal is lowered, its resistance decreases until it saturates at some residual value (blue). Some metals become superconducting at a critical temperature (green). However, in metals that contain a small fraction of magnetic impurities, such as cobalt-in-copper systems, the resistance increases at low temperatures due to the Kondo effect (red). (*b*) A system that has a localized spin embedded between metal leads can be created artificially in a semiconductor quantum-dot device containing a controllable number of electrons. If the number of electrons confined in the dot is odd, then the conductance measured between the two leads increases due to the Kondo effect at low temperature (red). In contrast, the Kondo effect does not occur when the dot contains an even number of electrons and the total spin adds up to zero. In this case, the conductance continuously decreases with temperature (blue).

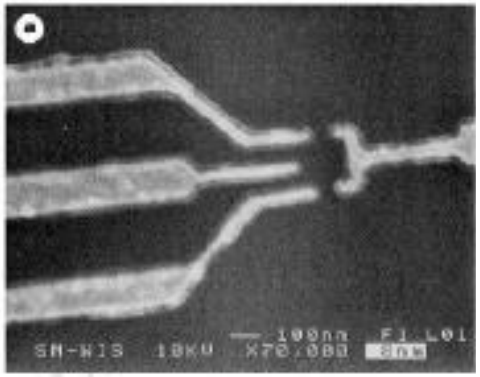
Physics World (January 2001) 33-38

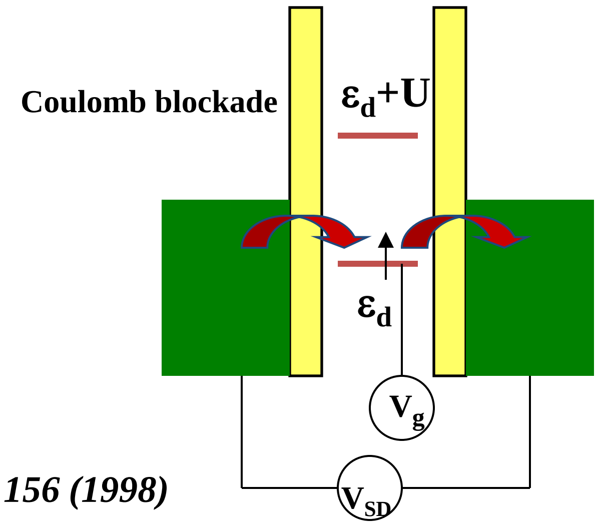
2 Spin flips

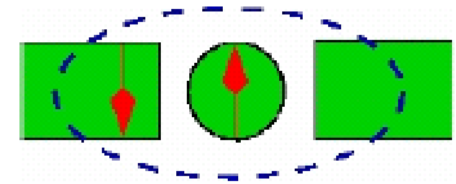


(*a*) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy ε_0 below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, *U*, while it would cost at least $|\varepsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden "virtual state" outside the impurity, and then be replaced by an electron from the metal. This can effectively "flip" the spin of the impurity. (*b*) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.

Kondo effect in quantum dot



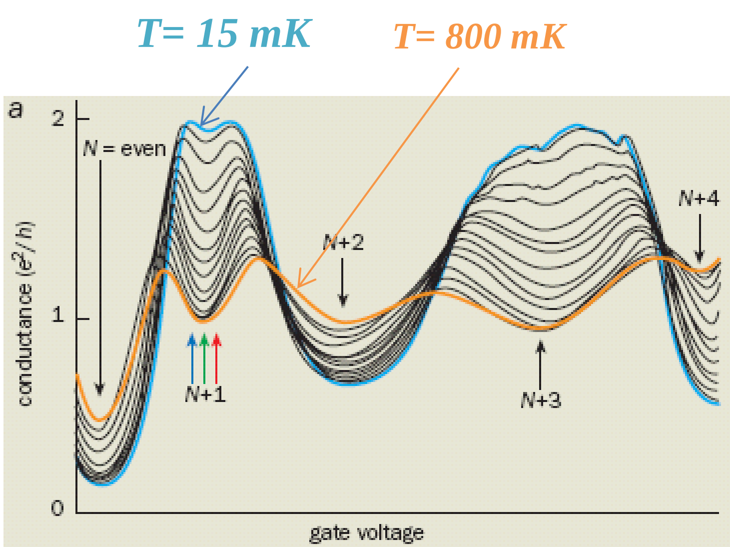


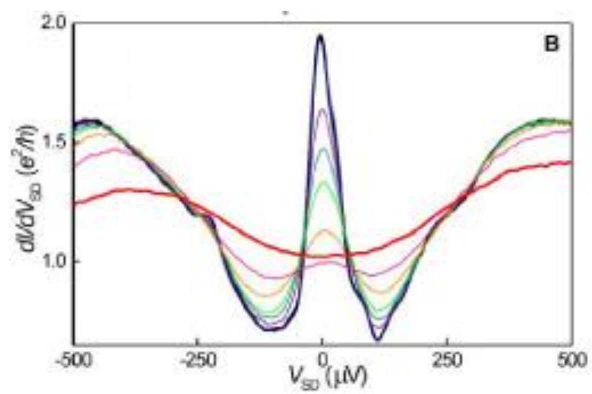


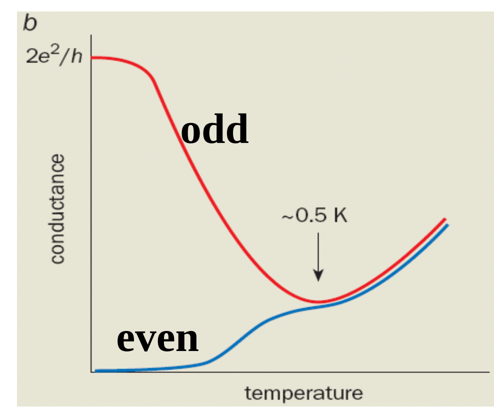
Kondo effect

Single quantum dot

Goldhaber-Gorden *et al. nature 391 156 (1998)*



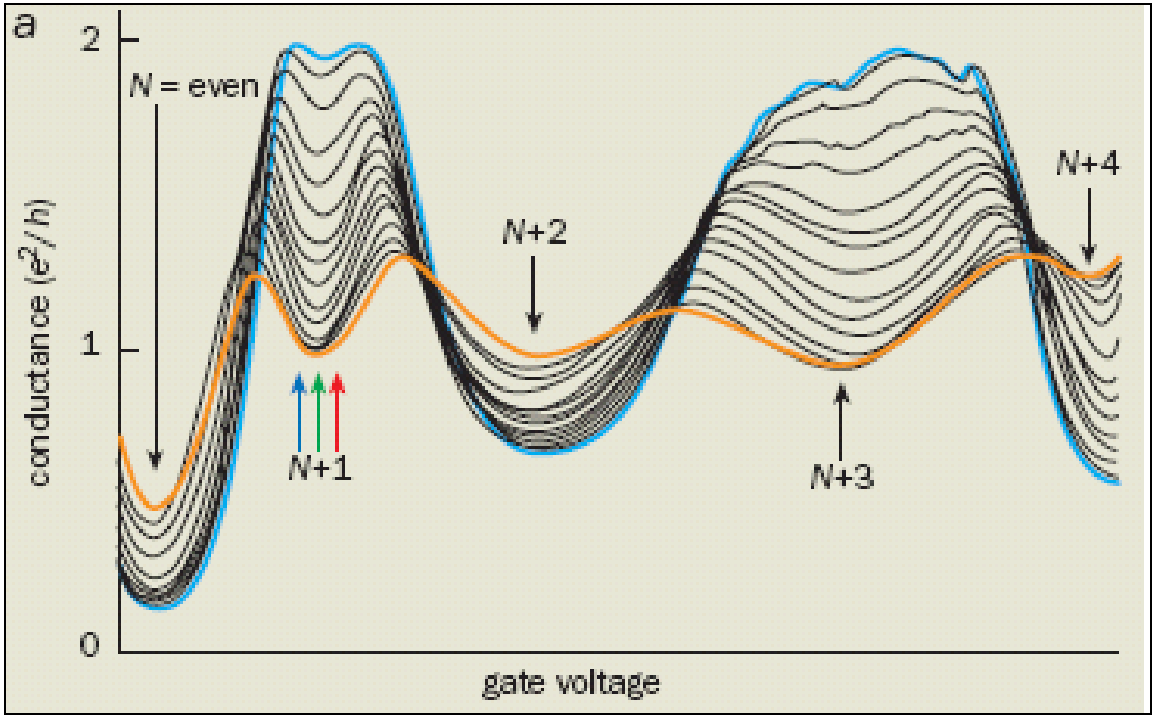


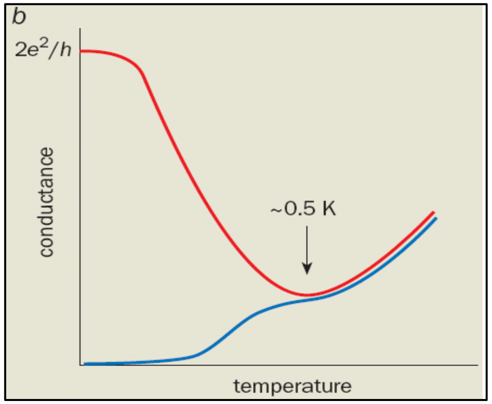


conductance anomalies

Glazman *et al. Physics world 2001*

L.Kouwenhoven *et al. science 289, 2105 (2000)* odd

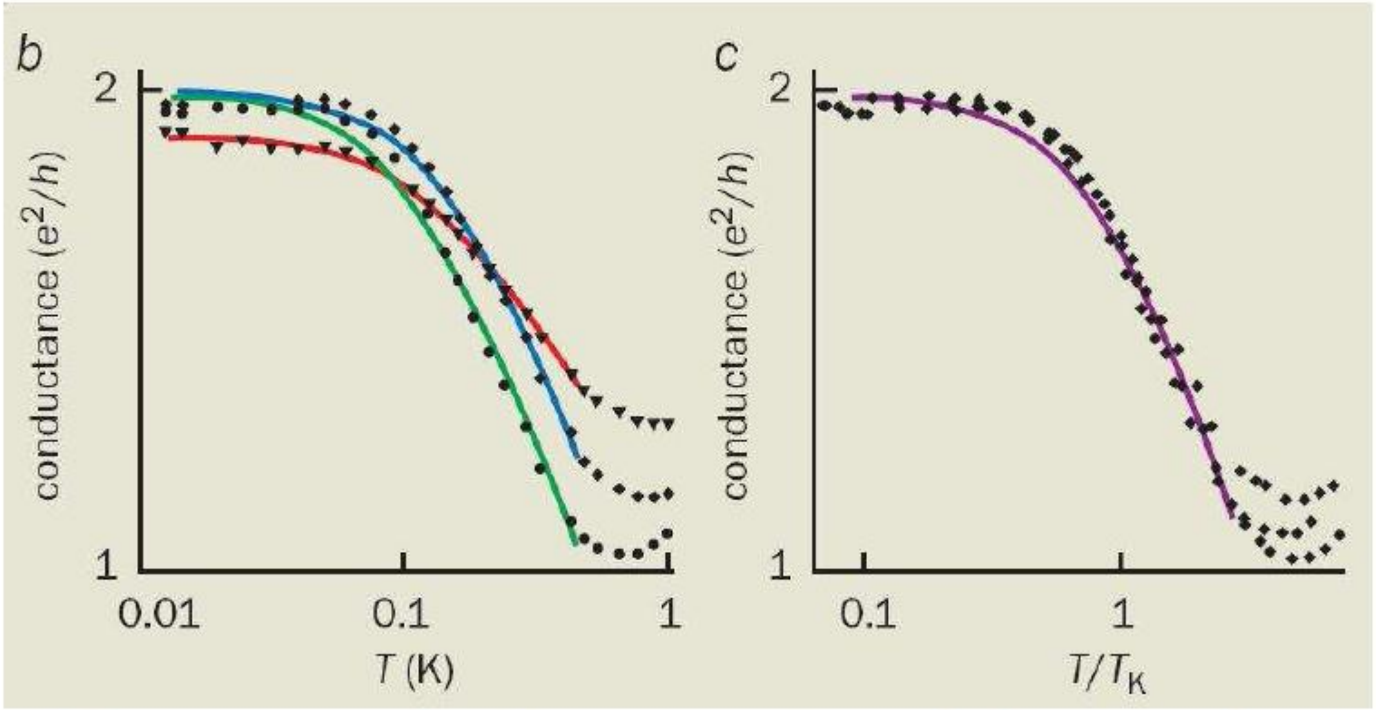




Glazman *et al. Physics world 2001*

L.Kouwenhoven *et al. science 289, 2105 (2000)*

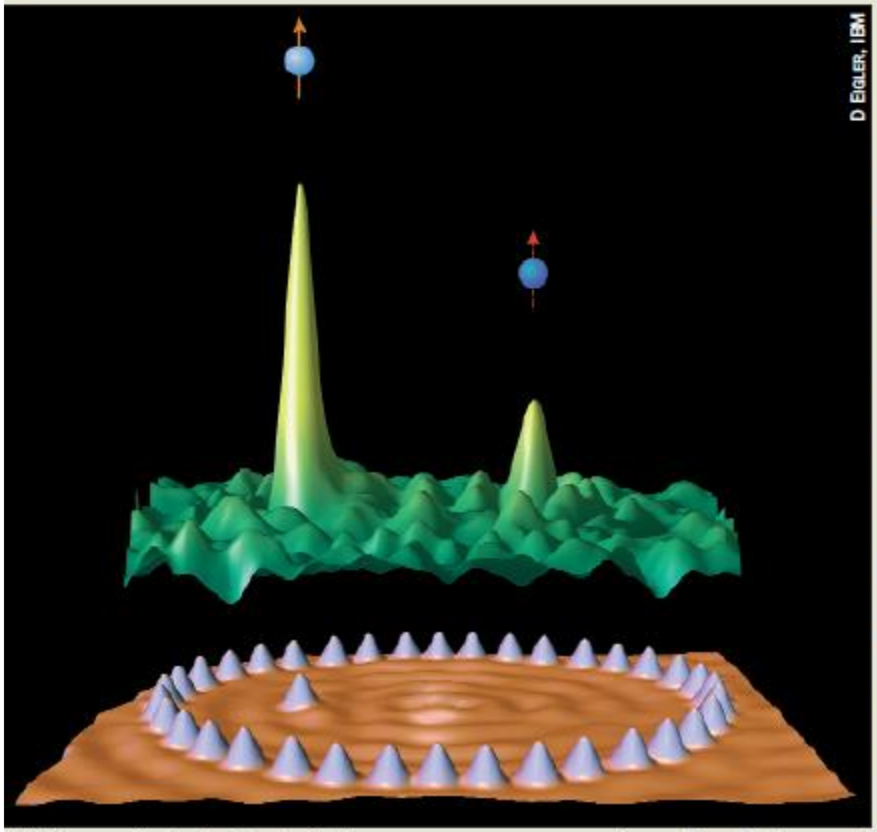
Quantized conductance vs temperature

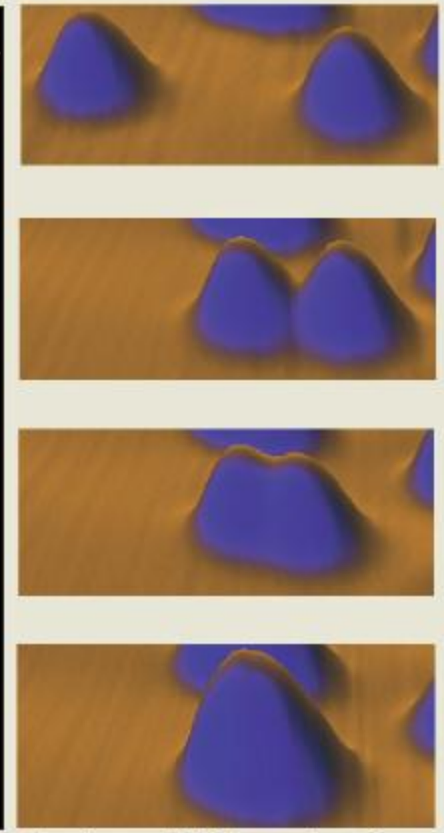


- **Universal relation between dimensionless conductance and temperature!**

Gate voltage is used to tune T_K; measurements at 50 to 1000 mK.

3 Single magnetic impurities under the microscope





(*a*) By manipulating cobalt atoms on a copper surface, Don Eigler and colleagues at IBM have placed a single cobalt atom at the focal point of an ellipse built from other cobalt atoms (bottom). The density of states (top) measured at this focus reveals the Kondo resonance (left peak). However, elliptical confinement also gives rise to a second smaller Kondo resonance at the other focal point (right) even though there is no cobalt atom there. (*b*) Meanwhile, Mike Crommie and co-workers have measured two Kondo resonances produced by two separate cobalt atoms on a gold surface (top). When two cobalt atoms are moved close together using an STM, the mutual interaction between them causes the Kondo effect to vanish (data not shown).

Question:

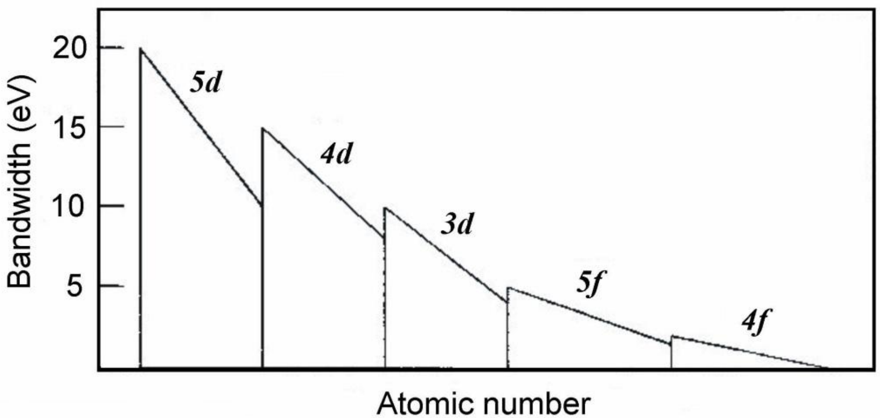
What happens if the Kondo lattice system is magnetically ordered?

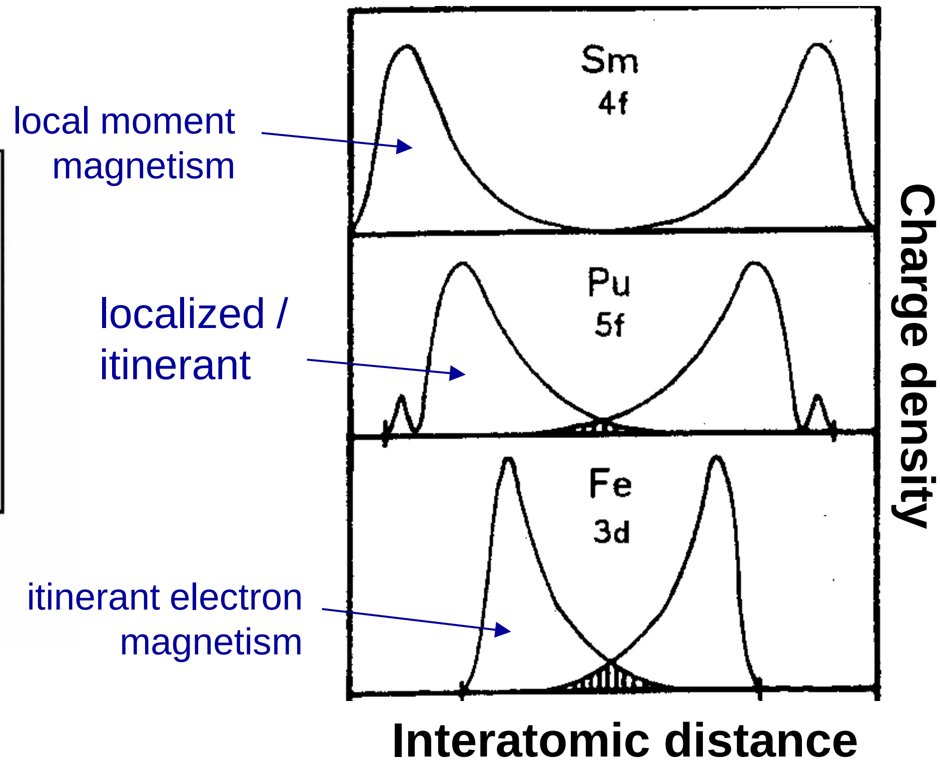
↓

First step

We consider magnetic interaction between localized 4f moments in a metallic systems!

Local versus Itinerant magnetic moments



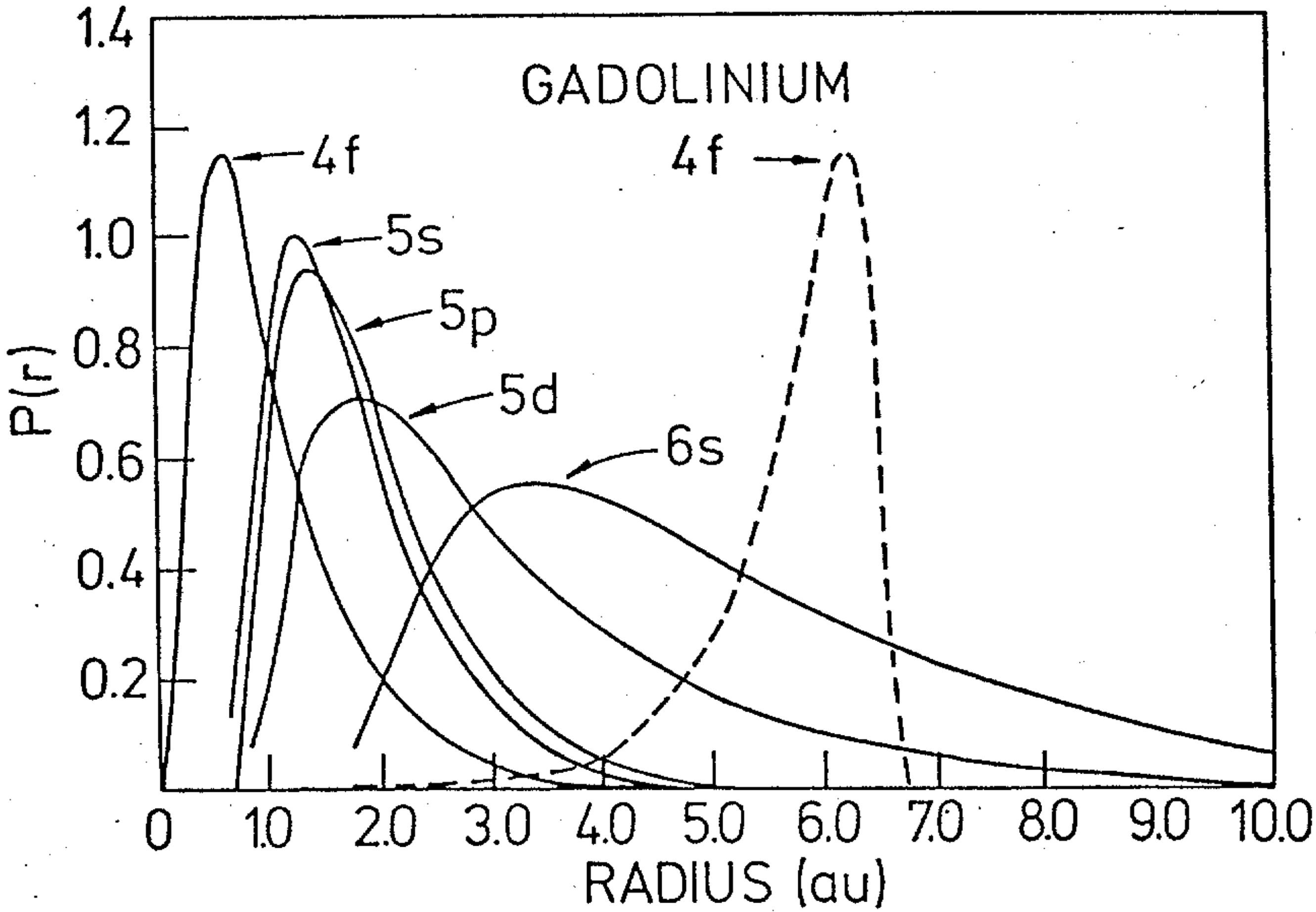


Bandwidth (W) of the metallic state

$$W(\text{Fe}) \approx 4 \text{ eV}$$
$$W(\text{Pu}) \approx 2 \text{ eV}$$
$$W(\text{Sm}) \leq 1 \text{ eV}$$

4f states are highly localized

No direct interaction possible!


GADOLINIUM
P(r)
RADIUS (au)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
4f
5s
5p
5d
6s
4f

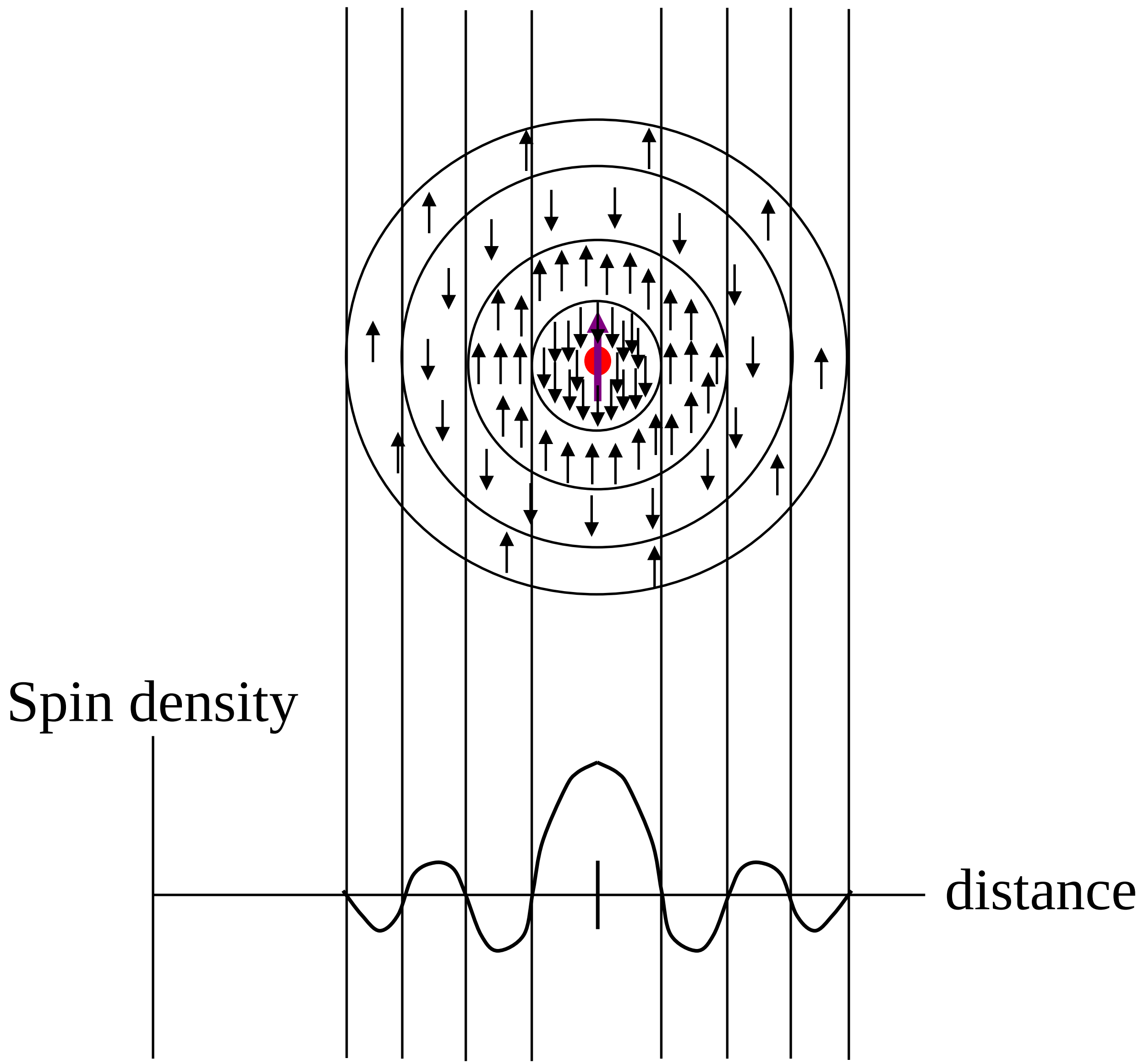
Exchange in Rare Earths (4f electrons)

- **Indirect exchange between 4f moments occurs**:

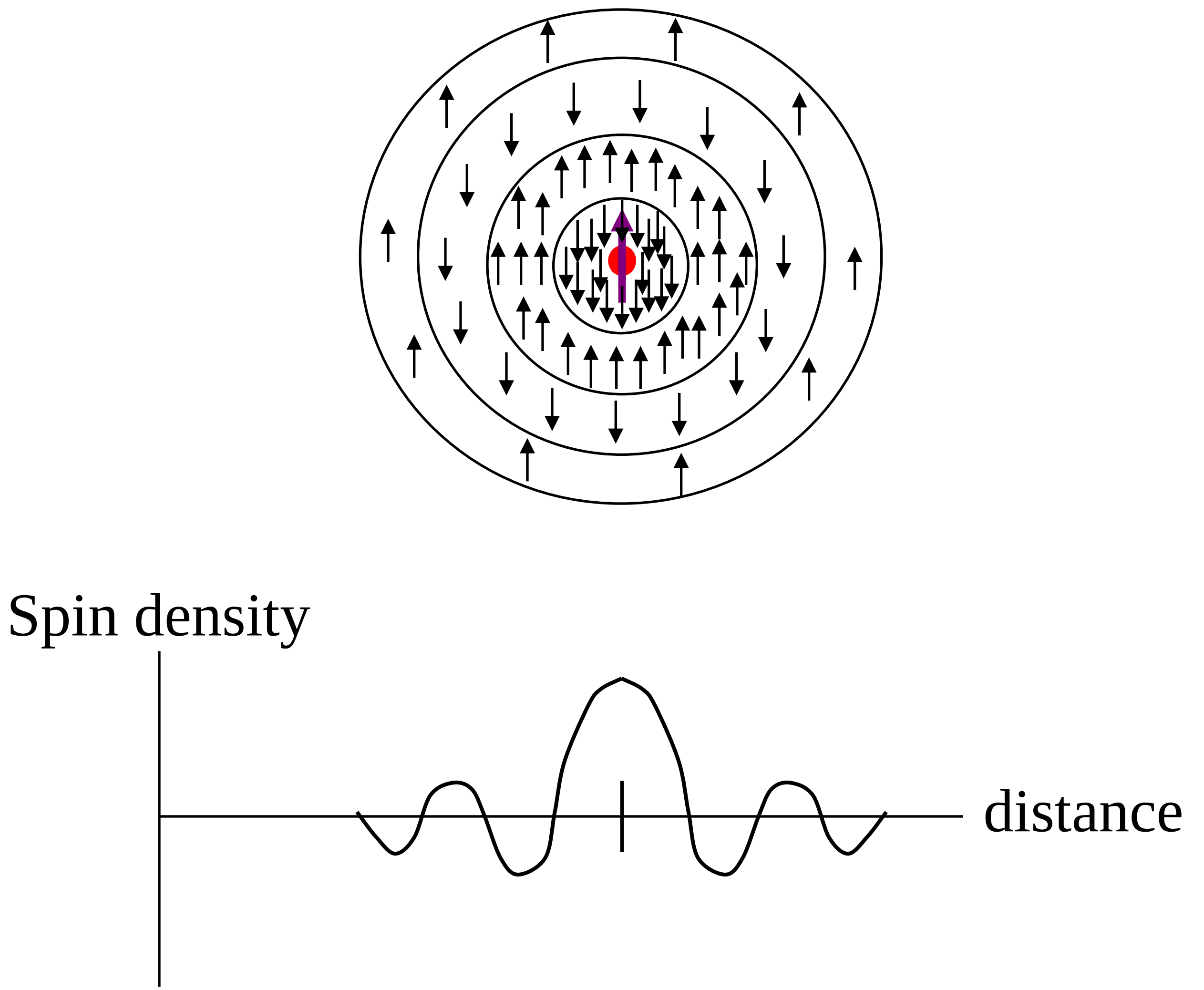
This type of exchange was first proposed by **R**uderman and **K**ittel and later extended by **K**asuya and **Y**osida to give the theory now generally know as the **RKKY** interaction.

It is the dominant exchange interaction in metals where there is little or no direct overlap between neighboring magnetic electrons

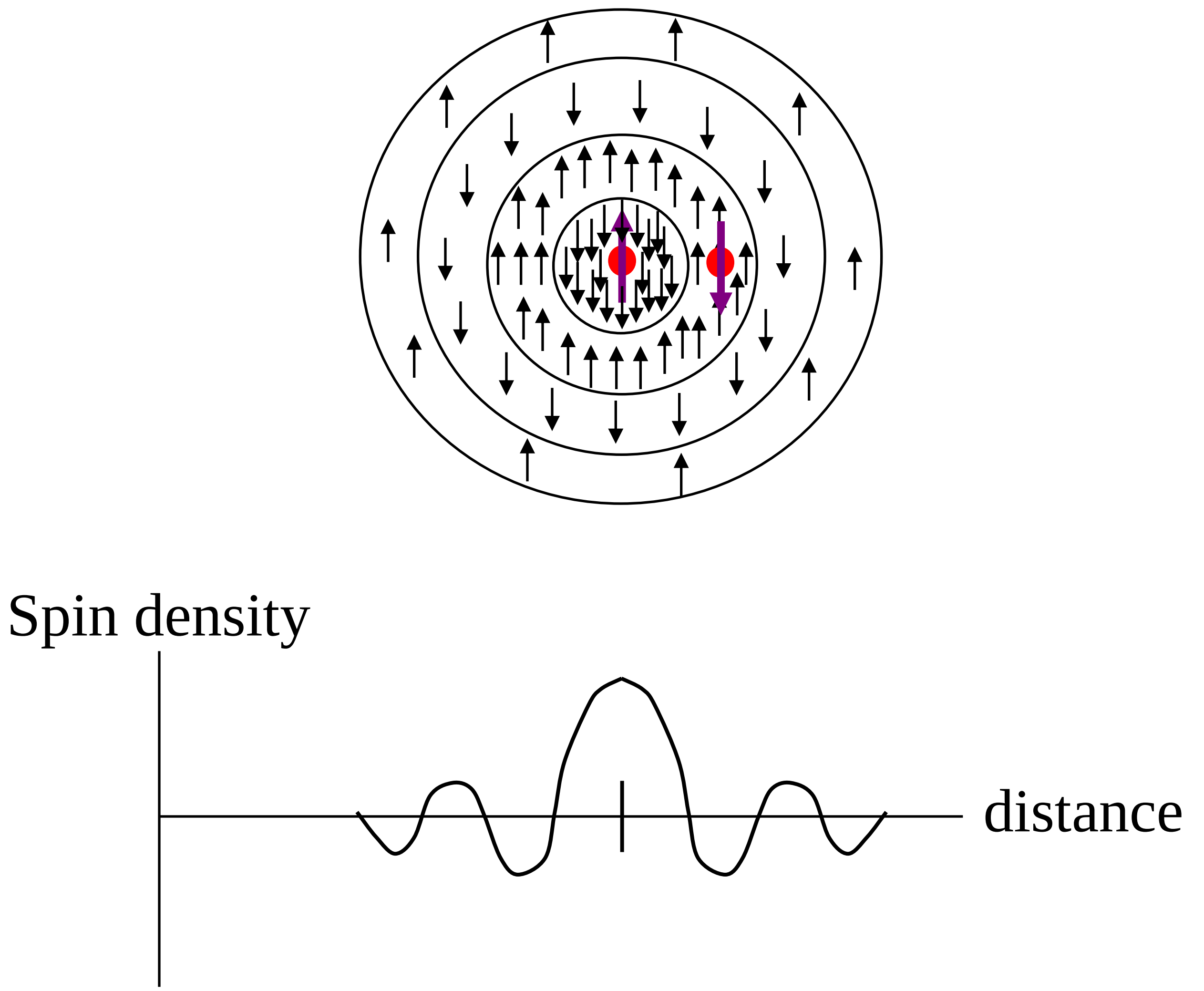
RKKY interaction



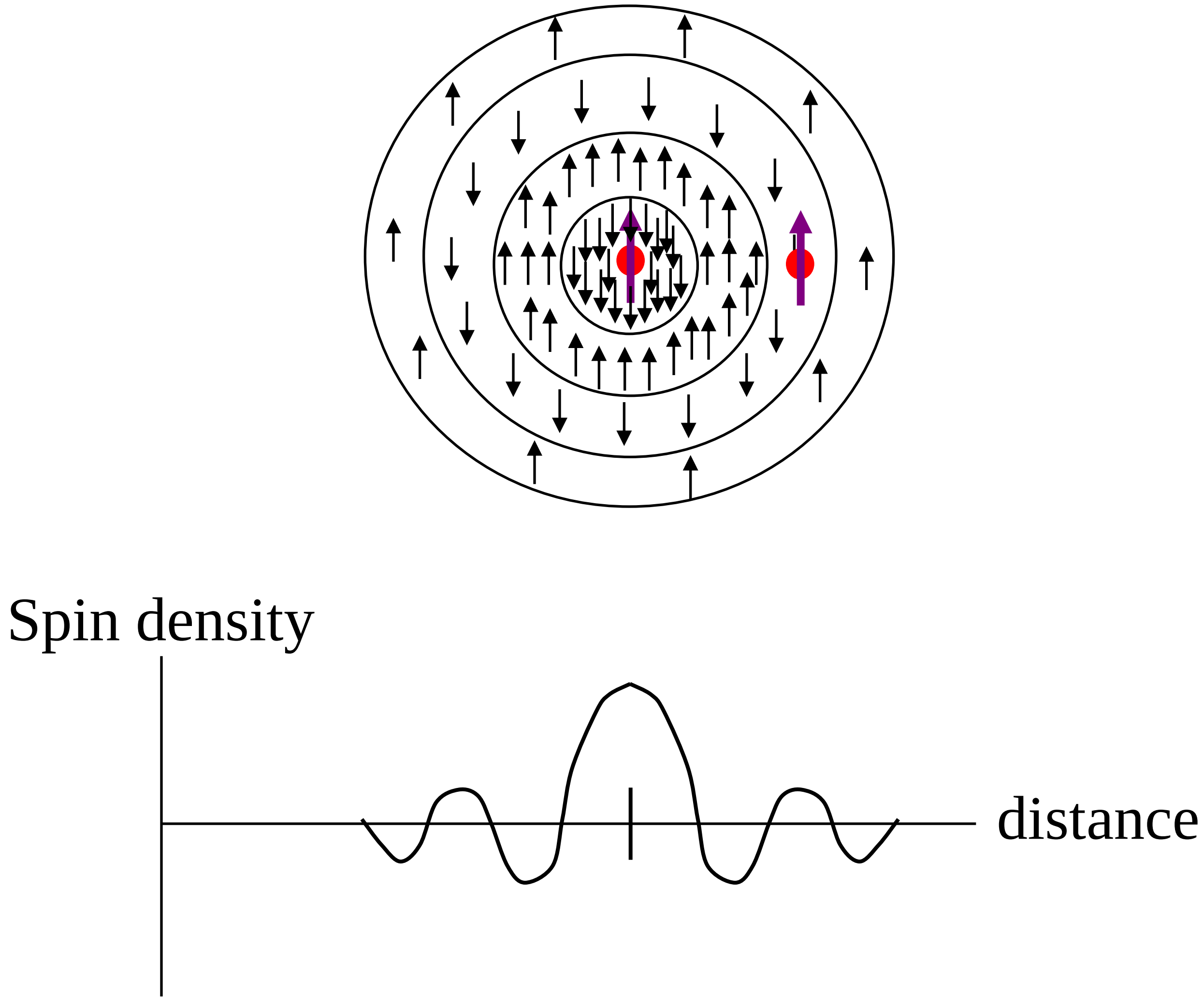
RKKY interaction



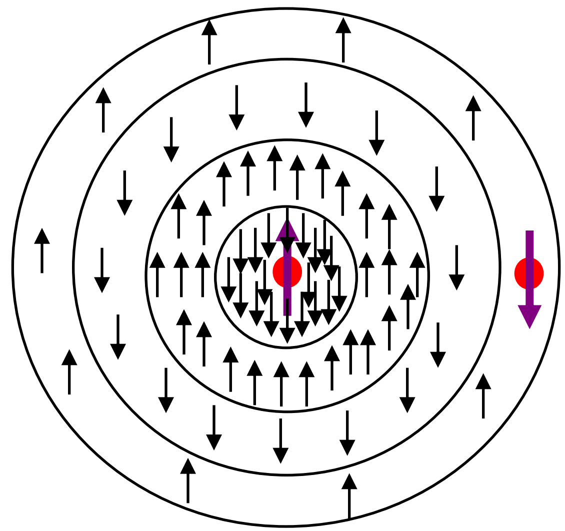
RKKY interaction

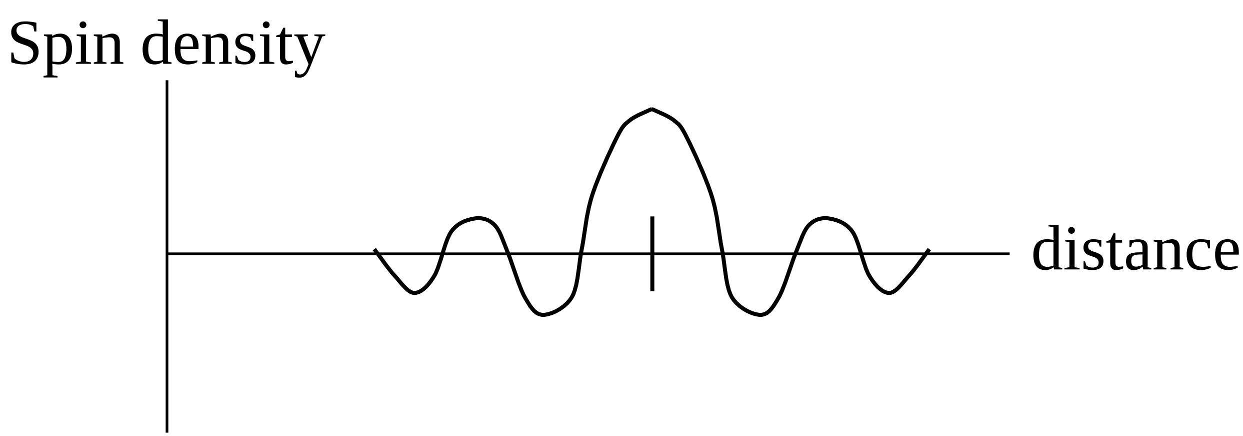


RKKY interaction



RKKY interaction

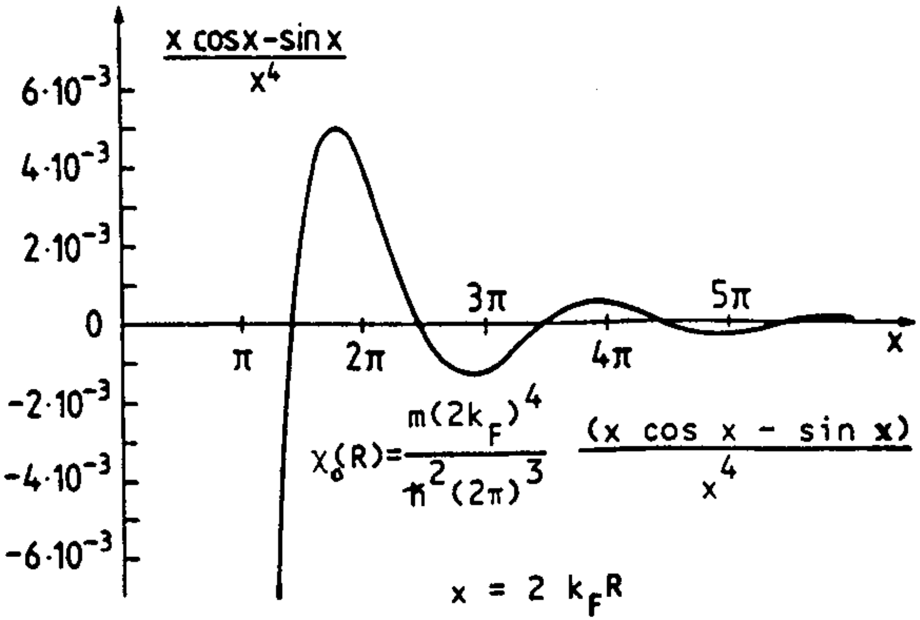




RKKY interaction: Description

Local moments (Spin S_i) in a sea of conduction electrons with itinerant spin s(r)

$$J(r) = 6\pi\, Z\, J\, N(E_F)\left[\frac{\sin(2k_F r)}{(2k_F r)^4} - \frac{\cos(2k_F r)}{(2k_F r)^3}\right]$$



Z	number of electrons / atom
J	s-d exchange interaction
$D(E_F)$	DOS at Fermi energy
k_F	Fermi momentum
r	distance between impurities

=> Oscillations of value and sign

P.H. Dederichs in: Magnetismus von Festkörpern und Grenzflächen, 24. IFF Ferienkurs, Jülich 1993

Question:

What happens if the Kondo lattice system is magnetically ordered?

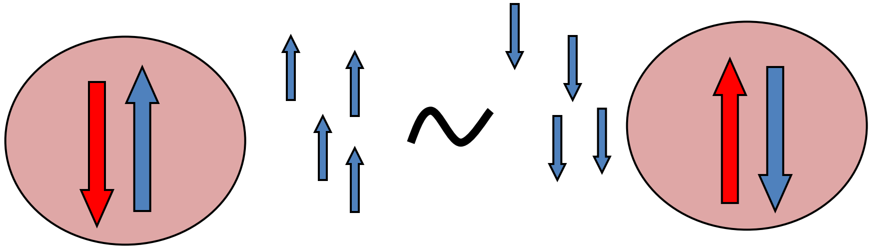
↓

second step

We consider in Kondo lattice the relatave strength of RKKY interaction and that of the Kondo effect

Theoretical description

Kondo-lattice-system: periodical arrangement of localized 4f- moments in a metallic matrix



Competition between:

Intrasite (on-site) interaction: Kondo-Effect

$\mathbf{E_K = k_B T_K}$

$\Rightarrow$ screening of the magnetic moments

$\Rightarrow$ **nonmagnetic ground state**

$T_K \sim \exp(-1/N(E_F)J)$

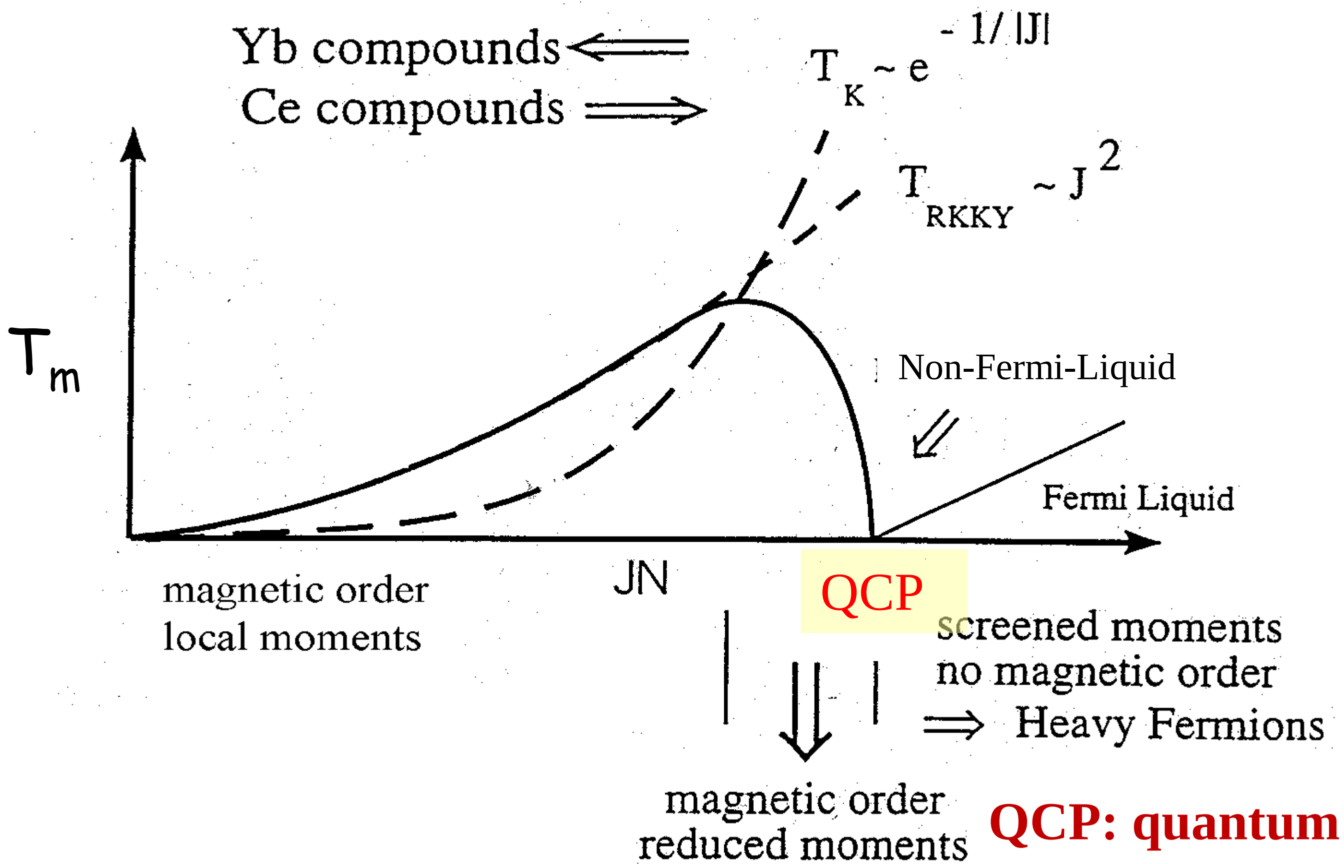
Intersite interaction: **RKKY**, $E_{RKKY} = k_B T_{RKKY}$

$\Rightarrow$ **long range magnetic order**

$T_{RKKY} \sim N(E_F)\, J^2$

J: interaction between f- and conduction electrons

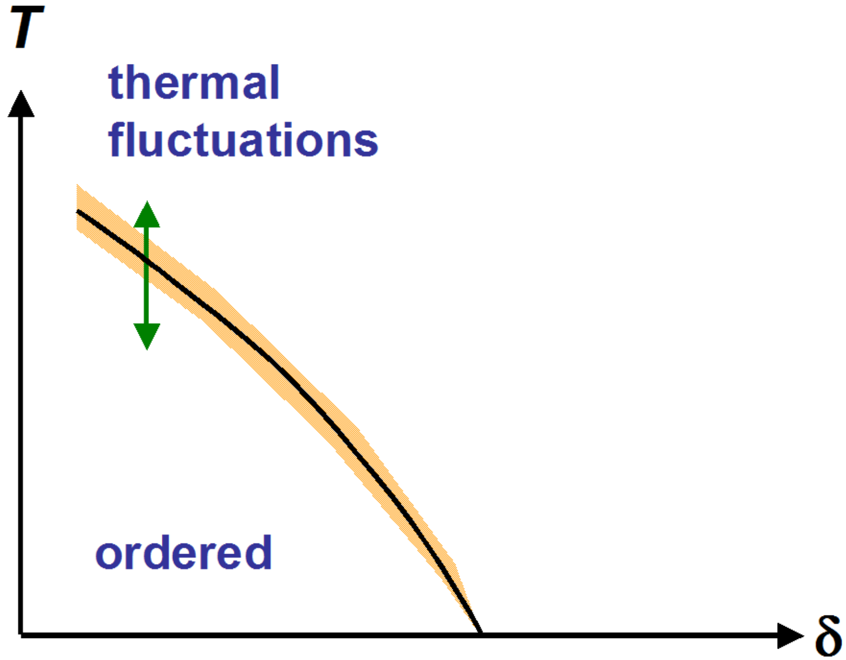
Doniach Model (Doniach 1977)



QCP: quantum critical point

The main result ... is that there should be a **second-order transition at zero temperature (at QCP)**, as the exchange coupling J is varied, between an antiferromagnetic ground state for weak J and a Kondo-like state in which the local moments are quenched.

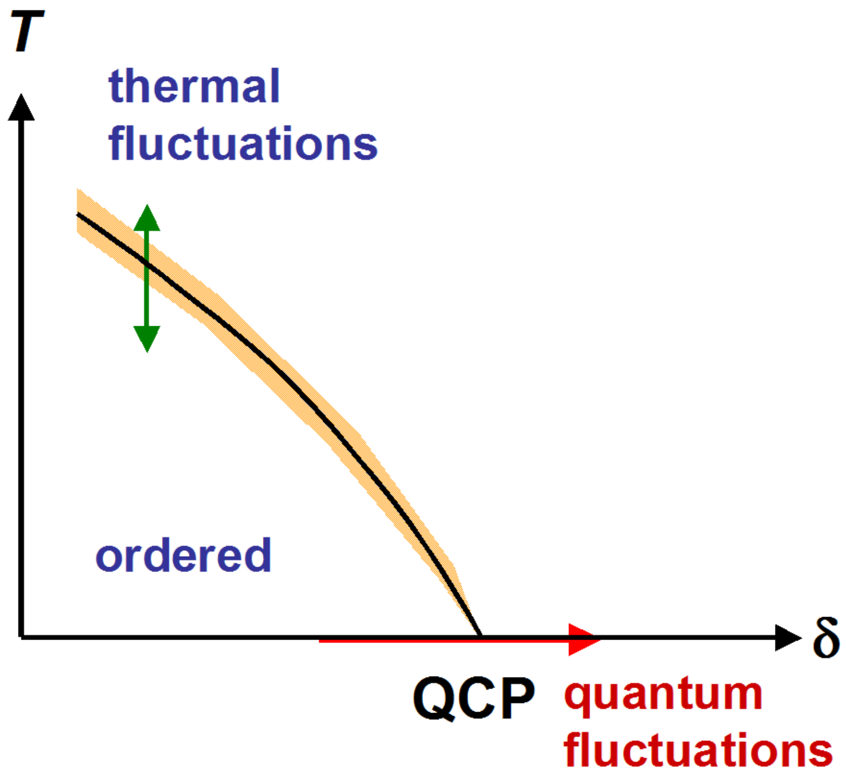
Quantum Phase Transitions



$\delta = p, B, \text{doping}$

- **classical phase transition: driven by thermal fluctuations**

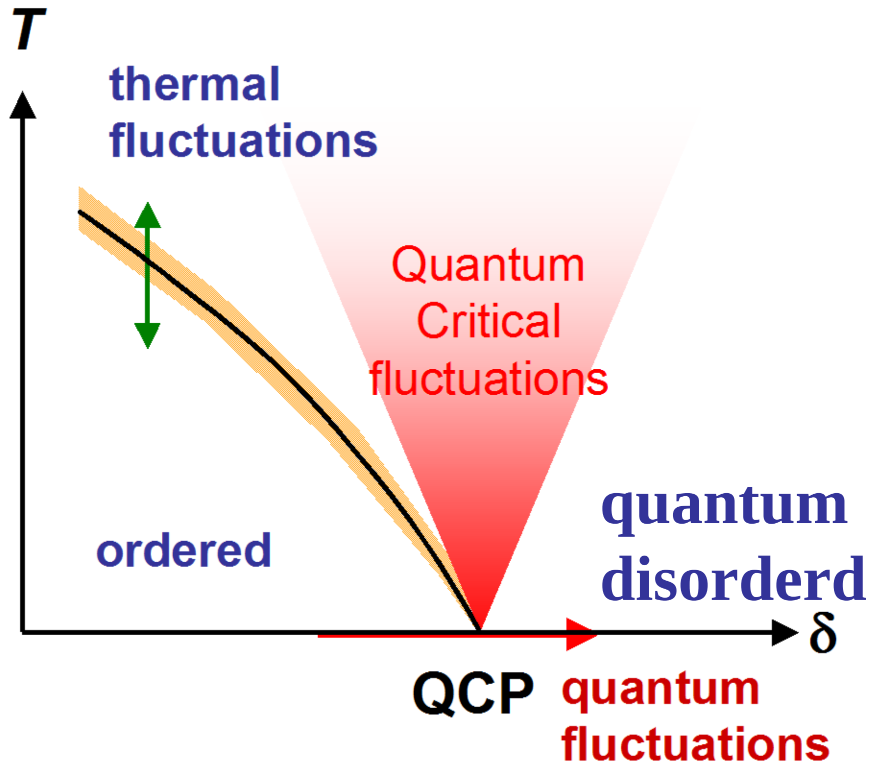
Quantum Phase Transitions



$\delta = p, B, \text{doping}$

- classical phase transition: driven by thermal fluctuations
- quantum phase transition: driven by quantum fluctuations

Quantum Phase Transitions



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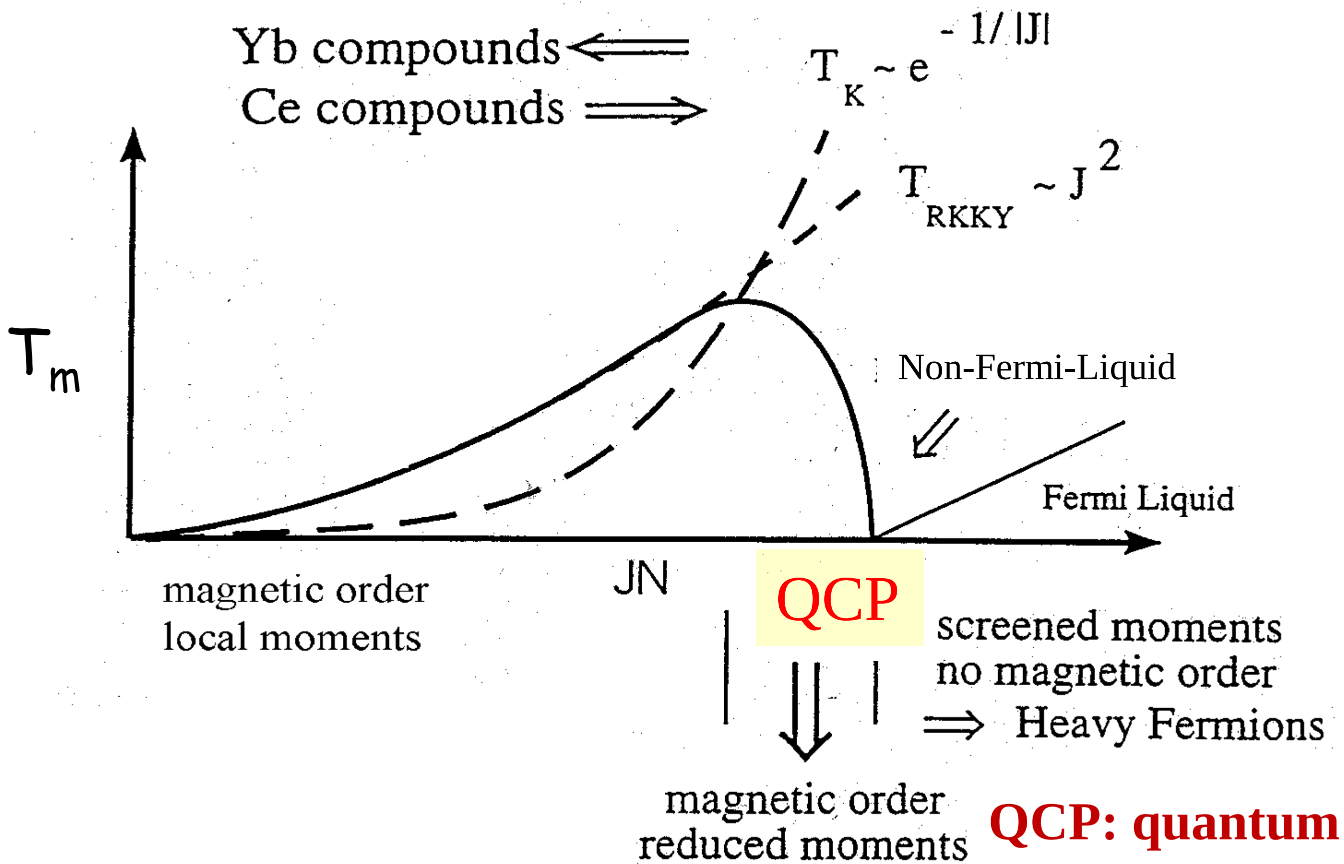

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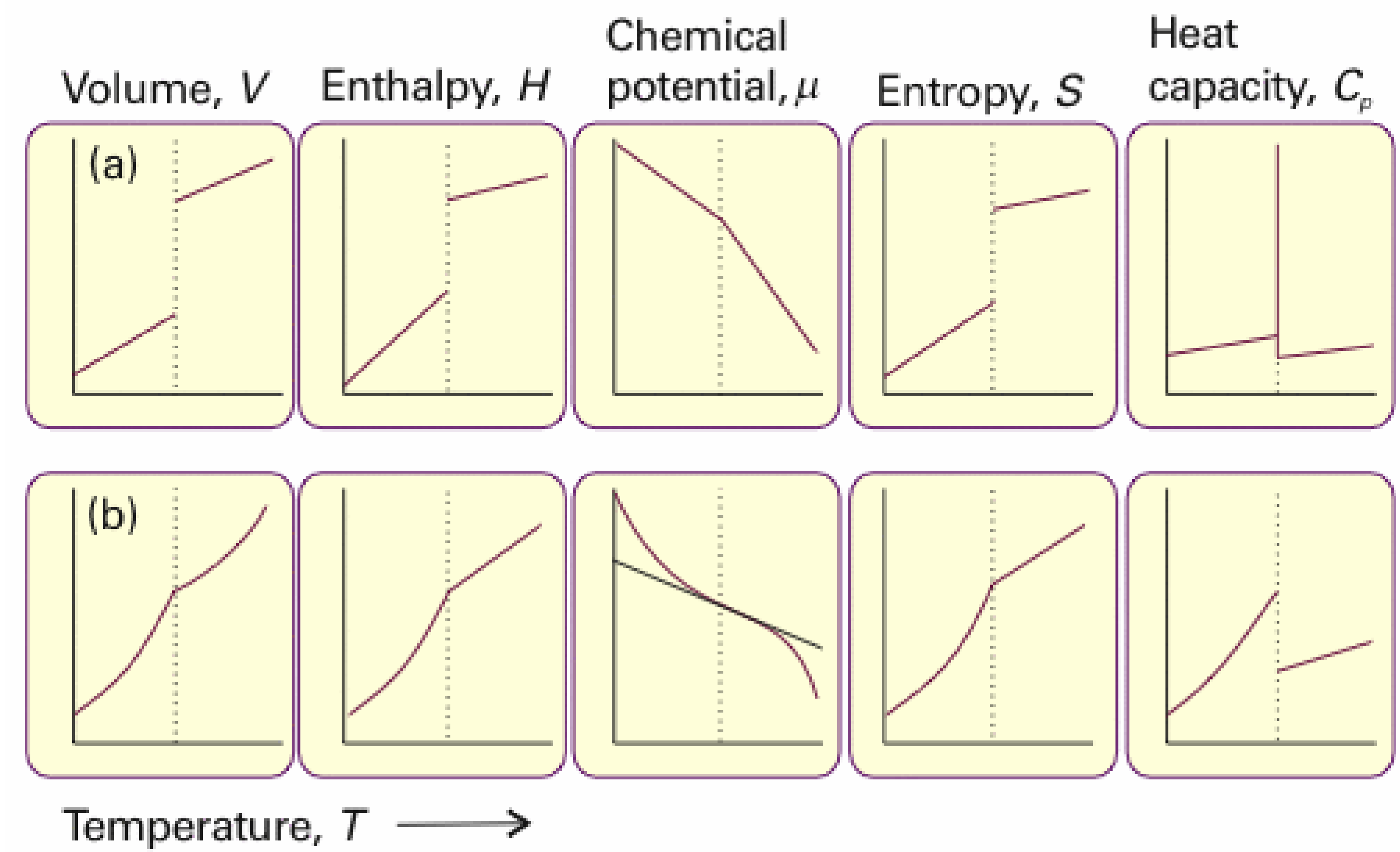
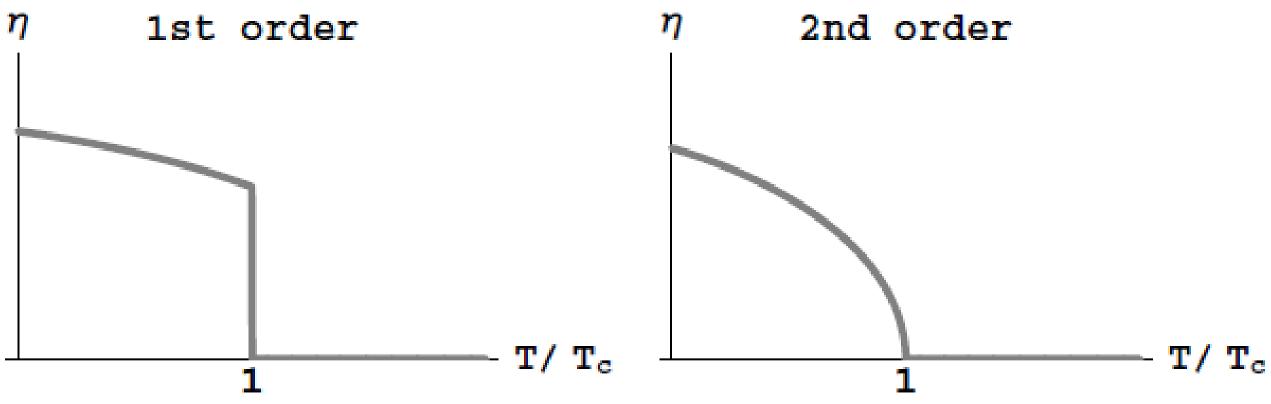
Types of Phase Transitions

First-order

second-order



Order parameter of a phase transition

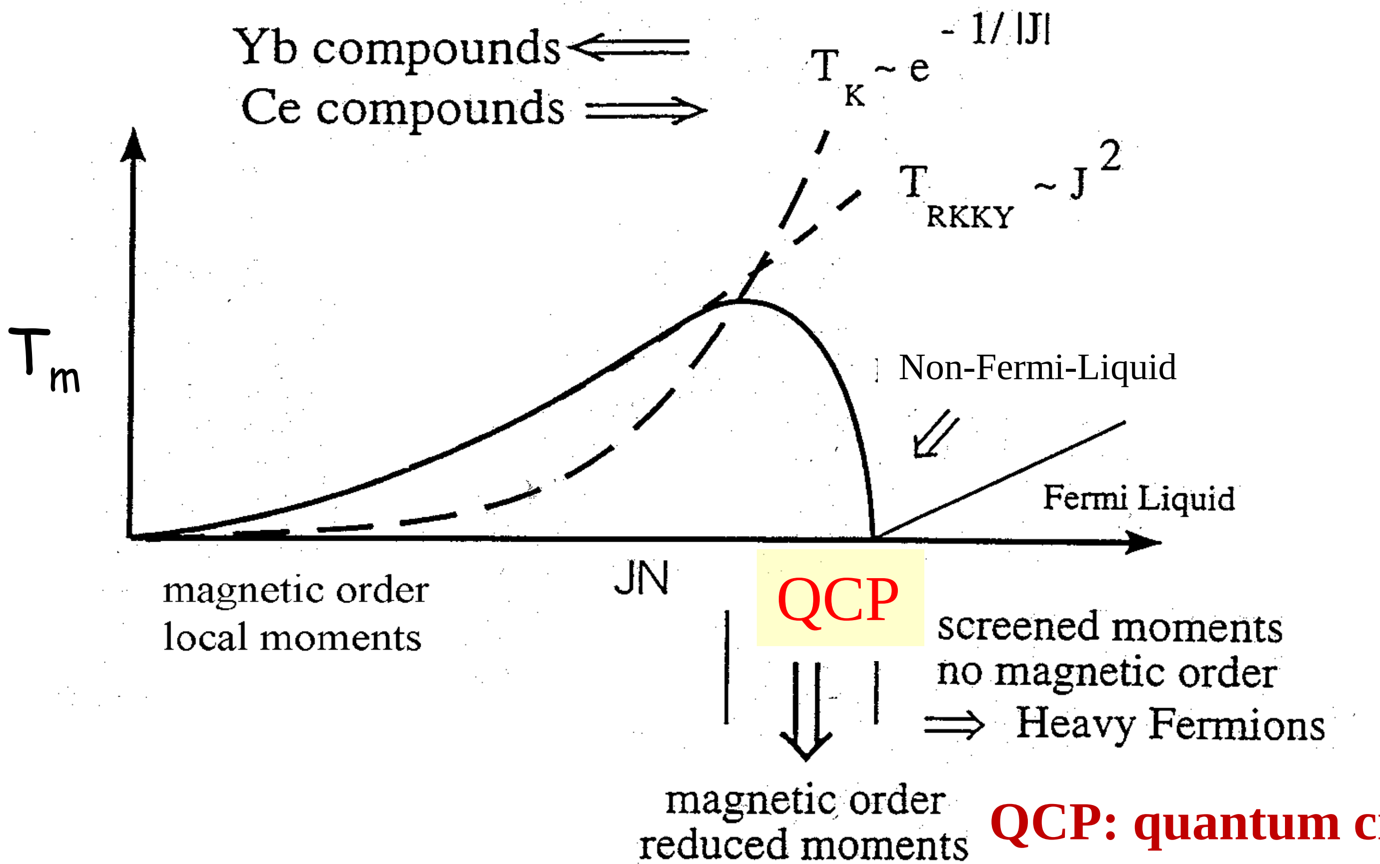


Partial list of transitions and order parameters

Transition	Order parameter
Liquid-gas	density
Ferromagnetic	magnetization
Ferroelectric	polarization
Superconductors	complex gap parameter
Siperfluid	condensate wave function
Phase Separation	concentration

L. P. Kadanoff *et al.*, Rev. Mod. Phys., 39 (1967), 395.

Doniach Model (Doniach 1977)



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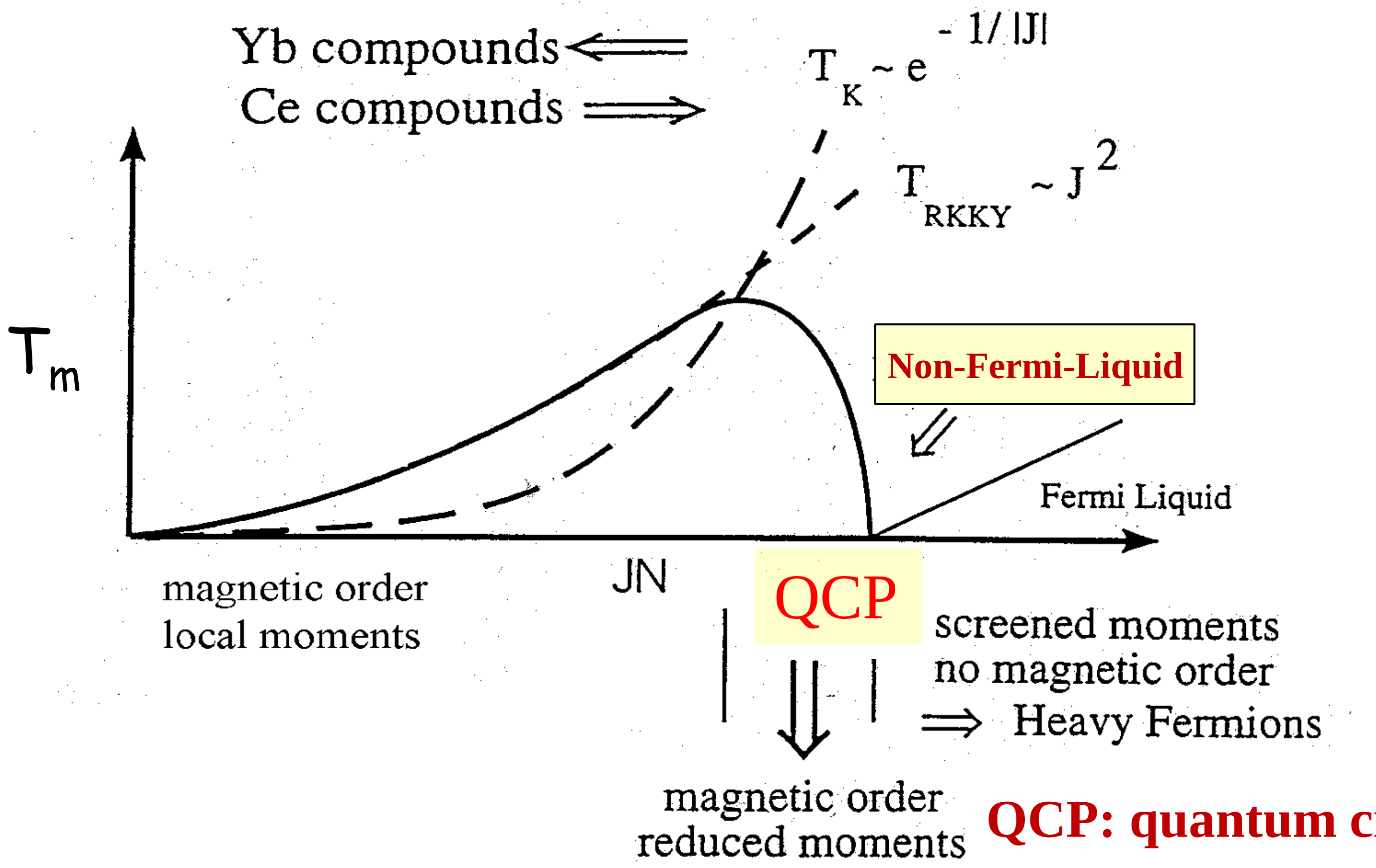


QCP: quantum critical point

Consequence: The nature of the ground state of the system strongly depends on the relative strength of RKKY interaction and Kondo effect.

Non-Fermi liquid behavior at magnetic quantum phase transitions

(here a brief discussion, for theoretical description see: H. v. Löhneysen et al., Rev. Mod. Phys. 79, 1015 (2007))

General aspects:

NFL behavior has been observed in U, Ce, Yb intermetallic compounds:

- chemically substituted: e.g. $Y_{1-x}U_xPd_3$, UCu_5Pd_x, ...
  (disordered systems)
- Stoichiometric: $(p=0) \Rightarrow UBe_{13}$, $CeCoIn_5$, $YbRh_2Si_2$, ...
  (proximity to QCP)

  $p > 0$ (pressure tuned) $\Rightarrow CeIn_3$, $CePd_2Si_2$, UGe_2, ...

$\Rightarrow$ different routes to NFL behavior!

* NFL behavior due to disorder: distribution of the Kondo temperature (distribution of J and or $D(E_F)$).
  (see Stewart, Rev. Mod. Phys. 73, 797 (2001)
  78, 743 (2006))
* multi-channel Kondo effect:
  f-electron spin is overscreened by the spins of conduction electrons
  $\Rightarrow$ antiferromagnetic superexchange interaction with electrons off the impurity site
* proximity to a QCP $\Rightarrow$ induced by quantum fluctuations

* Physical properties: weak power low, logarithmic divergences in
NFL T at low temperatures ($T \ll T_0$):

- electric resistivity, $\rho \sim a T^{\alpha}$ with $1 \leq \alpha \lesssim 1.6$ non-quadratic
- specific heat divided by T, $C/T \sim \ln(T_0/T)$ } diverging
- magnetic susceptibility, χ : $\chi \sim \ln(T_0/T)$ } diverging

Note that:

The appreciable T-dependence below T_0:

$\Rightarrow$ lower energy scale than Fermi liquid!

few comments to proximity to a QCP :

* At the QCP, the low temperature thermodynamics is determined by collective modes corresponding to fluctuations of the order parameter, rather than by single-fermion excitations as in FL => NFL properties arise.

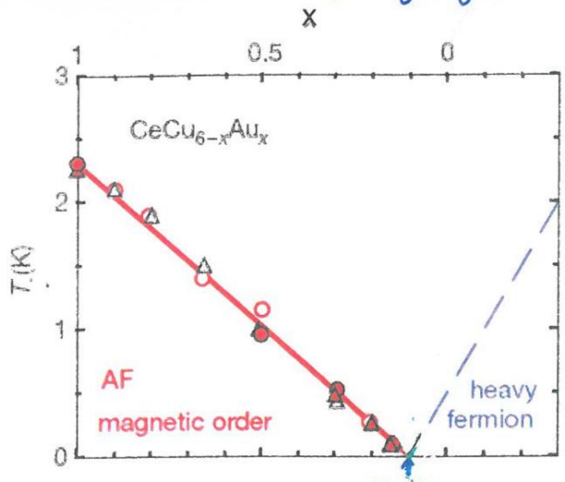
* NFL can also occur near quantum spin-glass or superconducting transitions.

* A quantum phase transition (like thermal or classical phase transition) is characterized by a diverging correlation length ξ and a diverging relation time ξ_T. However: (a) the critical fluctuations are quantum fluctuations rather than thermal fluctuations, and (b) contrary to classical critical point, the dynamic and static behavior of a QCP ar coupled together.

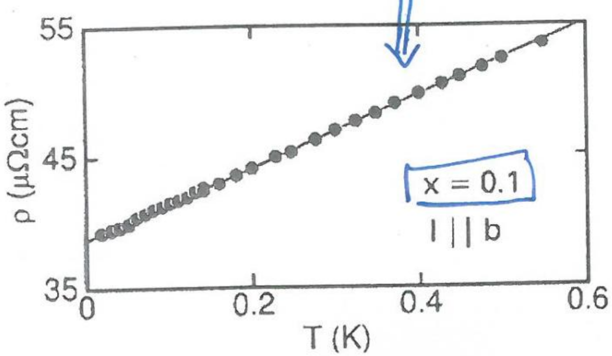
=) a system at a QCP will be affected in the same way by either a finite frequency or a finite temperature.

Type	Material	T^*	$T_c,\ x_c,\ B_c$	Properties	ρ	γ_n $mJmol^{-1}K^{-2}$
Metal	$CeCu_6$	$10K$	-	Simple HF Metal	T^2	1600
Super-conductors	$CeCu_2Si_2$	$20K$	T_c=0.17K	First HFSC	T^2	800-1250
	UBe_{13}	2.5K	T_c=0.86K	Incoherent metal→HFSC	$\rho_c \sim$ 150$\mu\Omega$cm	800
	$CeCoIn_5$	38K	T_c=2.3K	Quasi 2D HFSC	T	750
Kondo Insulators	$Ce_3Pt_4Bi_3$	$T_\chi \sim 80K$	-	Fully Gapped KI	$\sim e^{\Delta/T}$	-
	$CeNiSn$	$T_\chi \sim 20K$	-	Nodal KI	Poor Metal	-
Quantum Critical	$CeCu_{6-x}Au_x$	$T_0 \sim 10K$	$x_c = 0.1$	Chemically tuned QCP	T	$\sim \frac{1}{T_0} \ln\left(\frac{T_0}{T}\right)$
	$YbRh_2Si_2$	$T_0 \sim 24K$	$B_\perp$=0.06T $B_\parallel$=0.66T	Field-tuned QCP	T	$\sim \frac{1}{T_0} \ln\left(\frac{T_0}{T}\right)$
SC + other Order	UPd_2Al_3	110K	T_{AF}=14K, T_{sc}=2K	AFM + HFSC	T^2	210
	URu_2Si_2	75K	T_1=17.5K, T_{sc}=1.3K	Hidden Order & HFSC	T^2	120/65

$CeCu_{6-x}Au_x$: v. Löhneysen et al.:
physica B 223–224, 471 (1996)
J. Magn. Magn. Mater. 177–181, 12 (1998)



QCP
(x = 0.1)



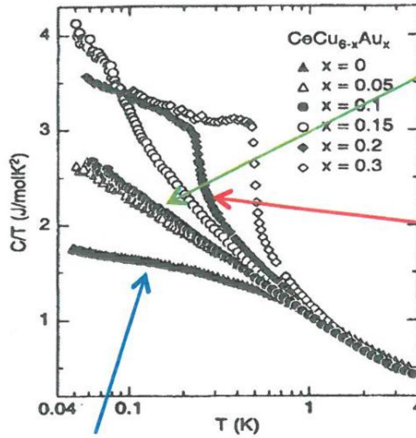
$\rho \sim T \Rightarrow$ NFL

Concentration dependence

Quantum critical

$C/T \propto \ln(T_0/T)$

$\Rightarrow$ NFL

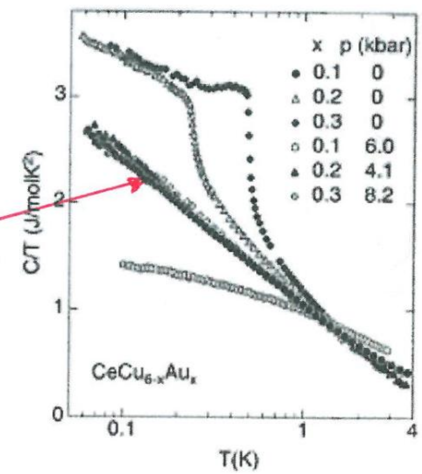


AF order

Heavy Fermion C/T = γ

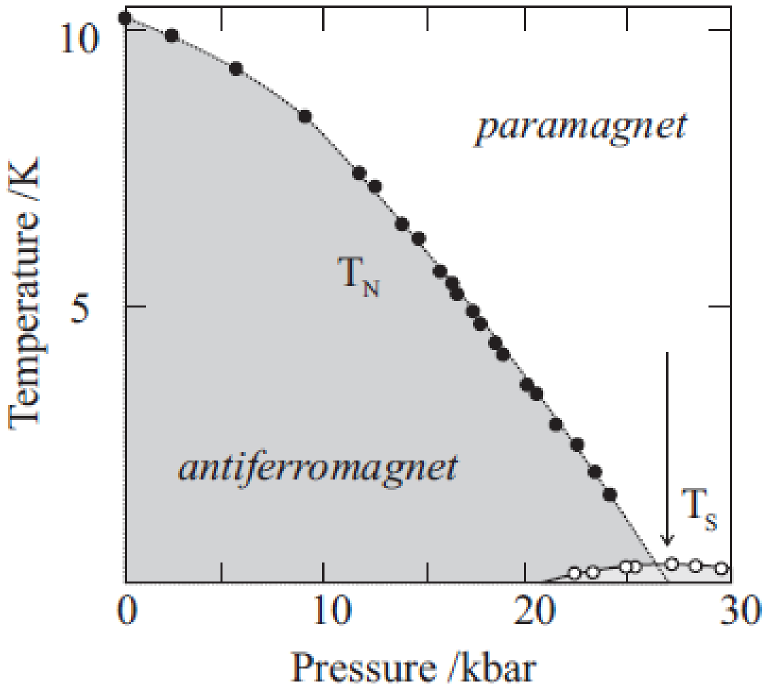
$\Rightarrow$ FL

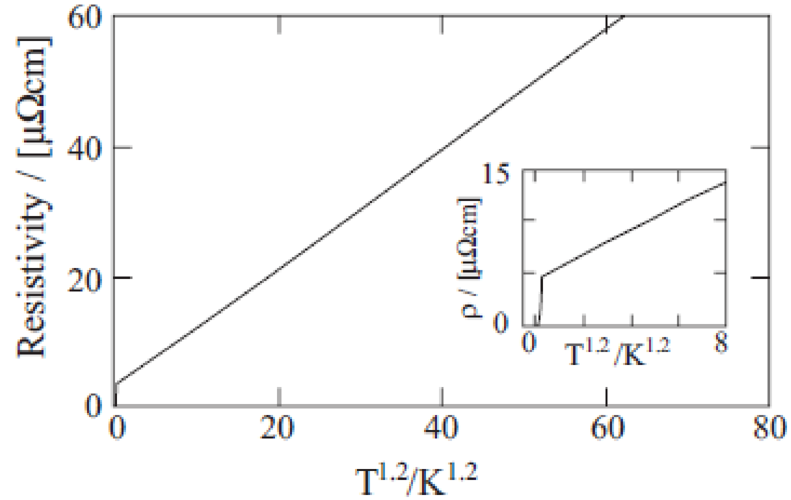
Pressure dependence



pressure-induced NFL
for x = 0.2 and x = 0.3
coincidence with
x = 0.1 at p = 0 !

Examples for non Fermi-liquid (NFL) behavior: CePd2Si2

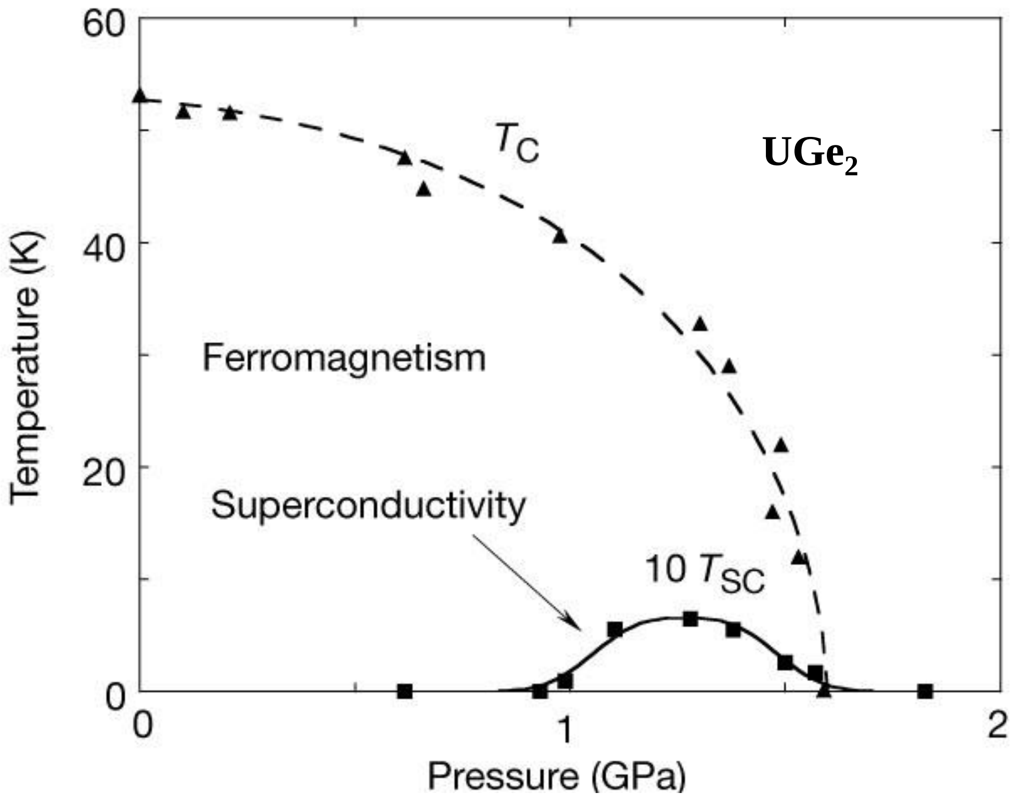


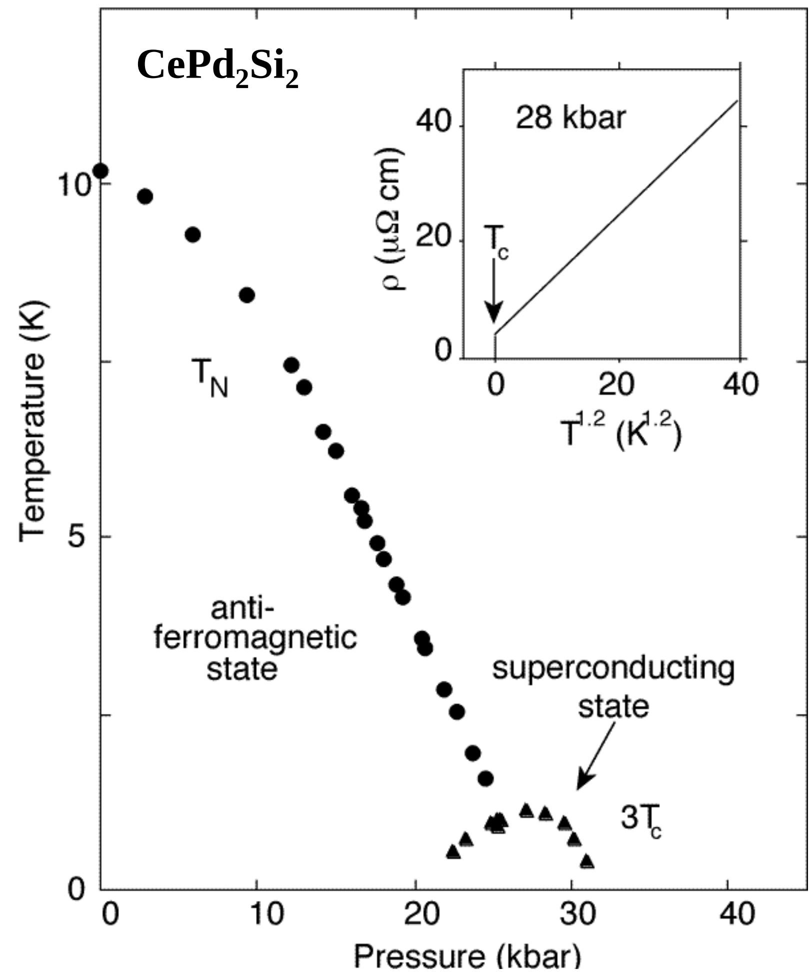


The resistivity of CePd2Si2 at the critical pressure (28 kbar). The observed temperature dependence, *T* 1.2, is seen over two decades of temperature. (Data after Grosche *et al.* 1996.)

CePd2Si2: a low temperature antiferromagnet. Under pressure the antiferromagnetism can be suppressed to zero temperature giving a quantum critical point. Not only non-Fermi liquid behavior but also there is a superconducting transition (after Julian *et al.* 1996, Mathur *et al.* 1998)

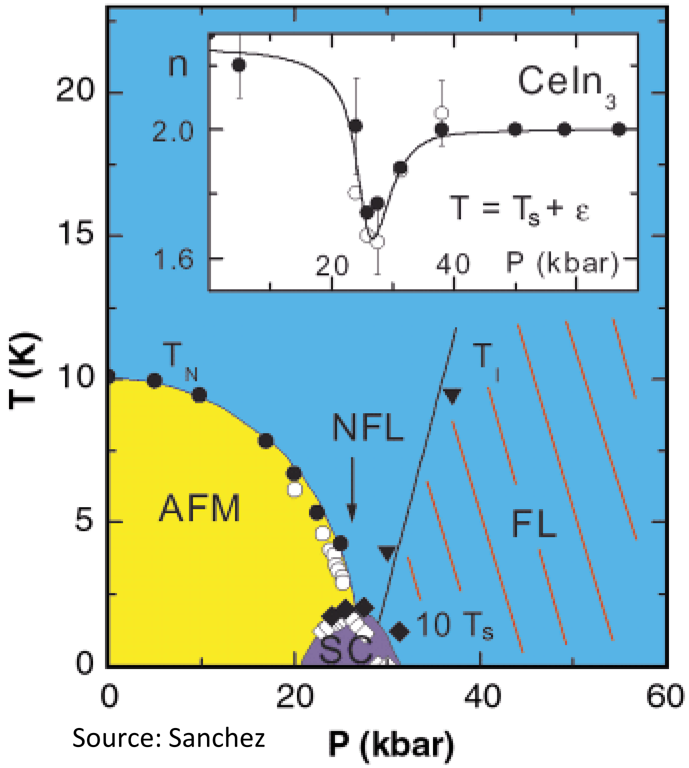
Heavy fermion systems





S. S. Saxena et al., Nature 406, 587 (2000) N. D. Mathur et al., Nature 394, 39 (1998)

Rich Phase Diagrams Exhibiting both NFL behavior and superconductivity.



Source: Sanchez

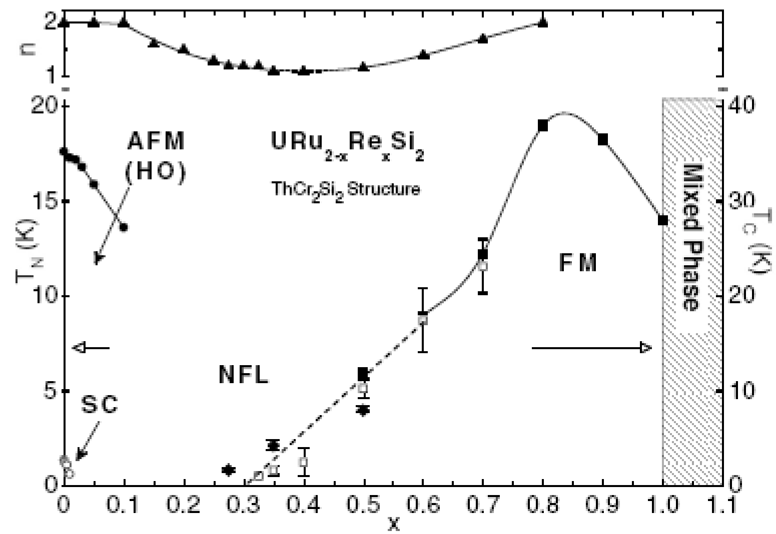
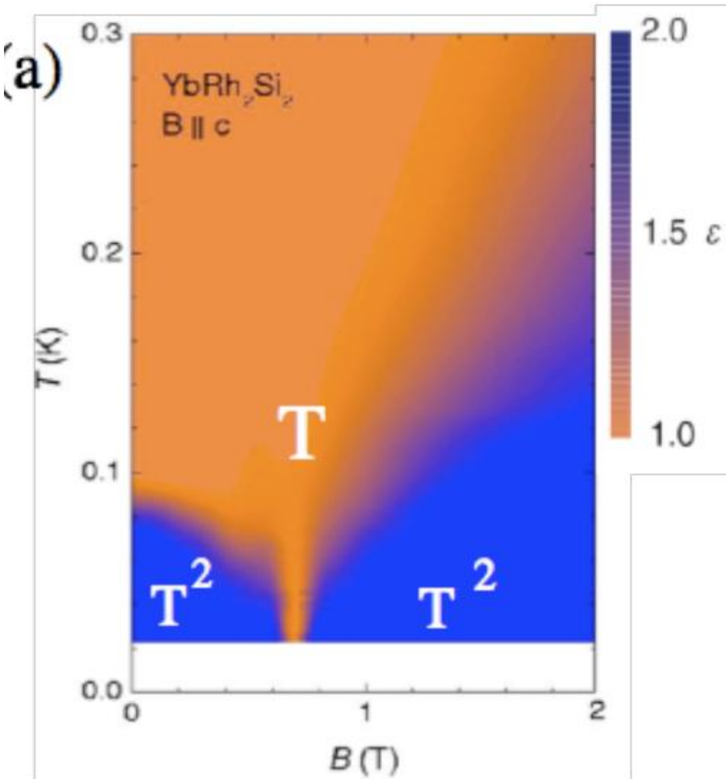


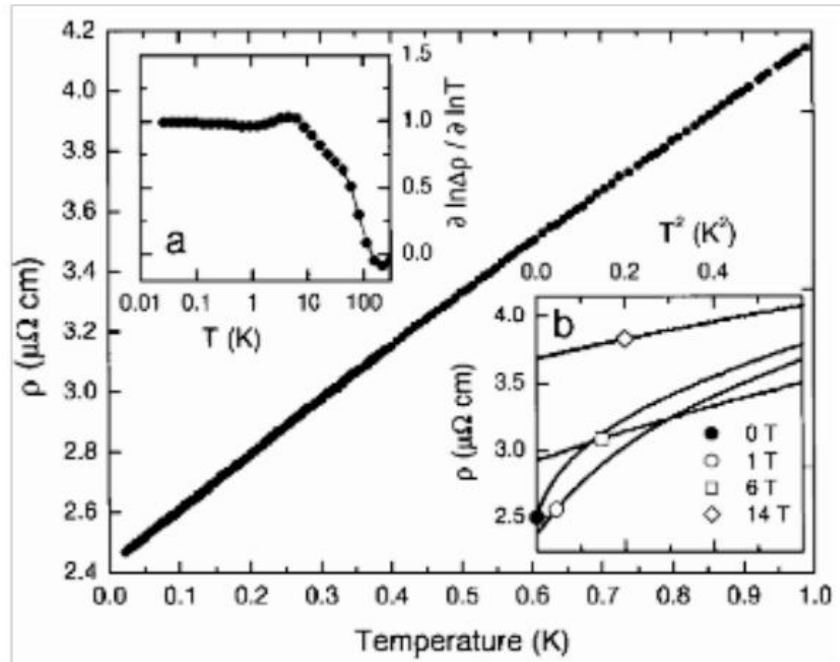
FIG. 1. The power-law exponent of $\rho(T)$, n, is shown in the top part of the diagram.

	$Y_{1-x}U_xPd$ (NFL)	**Fermi Liquid**
Heat Capacity	$C \sim -T\ln(T)$	$C = \gamma T$
Conductivity	$\rho \sim \rho_0 + AT^{1.1}$	$\rho = \rho_0 + AT^2$
Magnetic Susceptibility	$\chi_m \sim \alpha - \beta T^{1/2}$	$\chi_m = \beta$ Source: Seaman et al.

YbRh2Si2:

Field-induced quantum critical point





J. Custers *et al.*, Nature **424**, 524 (2003)

NFL behavior @ B = 0 in β-$YbAlB_4$

S. Nakatsuji *et al.*, Nature Phys. **4**, 603-607 (2008).

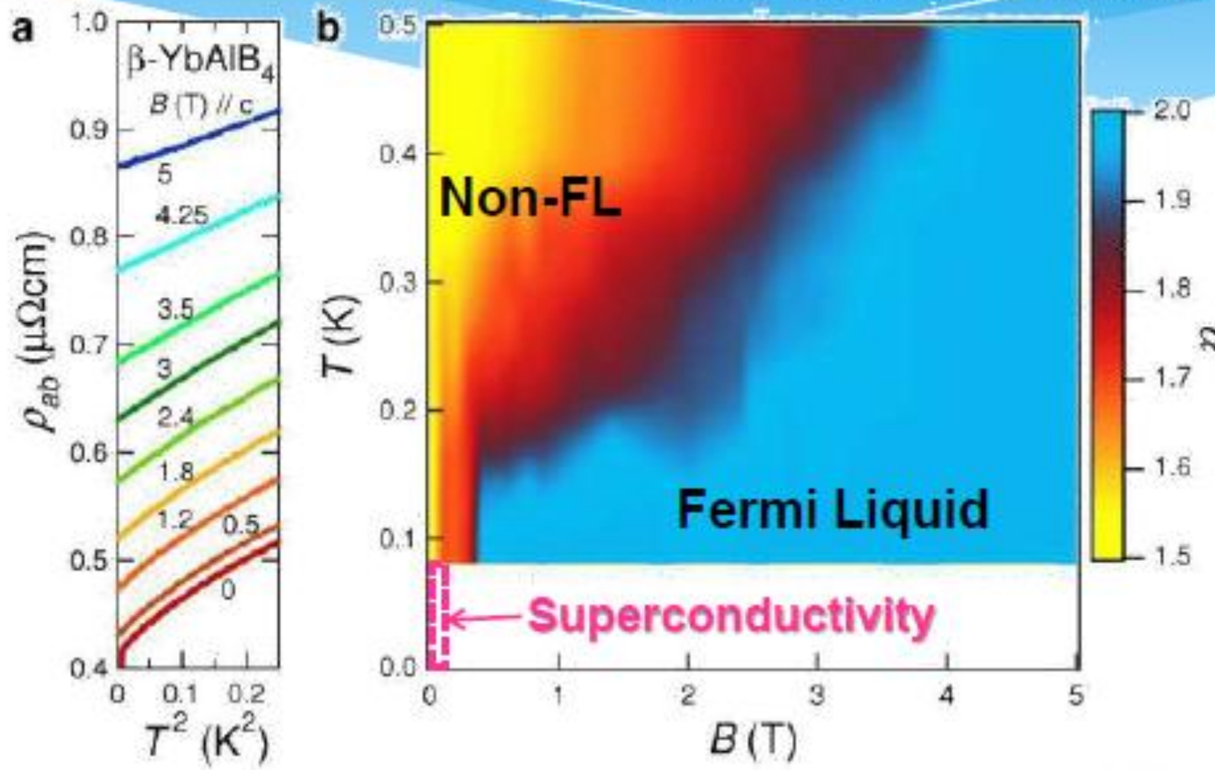
B = 0:
Non Fermi Liquid

- $\rho_{ab} \propto T^{1.5}$
- $M/H \propto T^{-0.5}$
- $C/T \propto \ln(T^*/T)$

↓

In magnetic field:
Fermi Liquid

- $\rho_{ab} \propto T^2$
- M/H = const.
- C/T = const.



$\Delta\rho = \rho(T) - \rho(0) = T^{\alpha}$

FL behavior is recovered in magnetic field.

QCP at B = 0 under P = 0 ?

Normally, we have to tune B or P or doping to approach QCP.

ex) $YbRh_2Si_2$, $CeCoIn_5$, $CeCu_{5.9}Au_{0.1}$, …